Chapter 9 – Welded Joints

Chapter Outline

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Welding symbols

- Finish symbol
- Contour symbol
- Root opening; depth of filling for plug and slot welds
- Size; size or strength for resistance welds
- Reference line
- Groove angle; included angle of countersink for plug welds
- Length of weld
- Pitch (center-to-center spacing) of welds
- Arrow connecting reference line to arrow side of joint, to grooved member, or both
- Field weld symbol
- Weld all around symbol
- Number of spot or projection welds
- Specification; process; or other reference
- Tail (may be omitted when reference is not used)
- Basic weld symbol or detail reference
Welding Symbols (cont.)

<table>
<thead>
<tr>
<th>Bead</th>
<th>Fillet</th>
<th>Plug or slot</th>
<th>Groove</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Square</td>
</tr>
</tbody>
</table>
Welding Symbols (cont.)
Fillet Welds
Circular Fillet Weld

Circle indicates that weld goes around full circumference
Butt Joint

(a) Tensile loading

(b) Shear loading
Stress Transfer in Welded Joints

- High shear stresses
- Multiaxial stress state
- High tensile stress
Stress Distribution in Welded Joint (cont.)

Welded joint

Finite element model
Stress Distribution in Welded Joint (cont.)

Finite Element Mesh
Stress Distribution in Welded Joint (cont.)

Tensile stress distribution

Shear stress distribution
Stress Distribution in Welded Joint (cont.)

von Mises effective stress distribution
Analysis of Transverse Fillet Weld

\( F_n, F_s \) – normal and shear force on section defined by the angle \( \theta \)
Average Stress in Transverse Fillet Weld

Average shear and normal stress

\[
\tau = \frac{F}{hl} \left( \sin \theta \cos \theta + \sin^2 \theta \right)
\]

\[
\sigma = \frac{F}{hl} \left( \cos^2 \theta + \sin \theta \cos \theta \right)
\]

von Mises effective stress

\[
\sigma' = \sqrt{\sigma^2 + 3\tau^2}
\]
Average Stress in Transverse Fillet Weld (cont.)

Maximum von Mises stress occurs at $\theta = 62.5^\circ$ where

$$\tau = 1.196 \frac{F}{hl}, \quad \sigma = 0.623 \frac{F}{hl}, \quad \sigma' = 2.16 \frac{F}{hl}$$

Maximum shear stress occurs at $\theta = 67.5^\circ$ where

$$\tau_{\text{max}} = 1.207 \frac{F}{hl}, \quad \sigma = 0.5 \frac{F}{hl}$$

Note that these formulas assume uniform stress across the weld.
Actual Stress Distribution in Transverse Fillet Weld

Note: stresses are highly non-uniform
Design of Fillet Weld

Use simple and conservative models:
- Consider shear load on throat of the weld
- Ignore normal stress acting on throat

\[ \tau_{\text{max}} = \frac{F}{0.707hl} = 1.414 \frac{F}{hl} \]
Approach to Designing Weld Joints

Compute primary shear stress due to external forces
Compute secondary shear stresses due to torsion and bending effects
Determine strengths of both parent metal and deposited weld material
Determine permissible loads for both parent metal and deposited weld material
Welded Joint Loaded in Shear

\[ \tau = \frac{F}{0.707 hl} = \frac{1.414 F}{hl} \]
Stresses in Welded Joints in Torsion

Primary Shear

\[ \tau' = \frac{V}{A} \]

where \( A \) is the throat area of all of the welds

Secondary Shear

\[ \tau'' = \frac{Mr}{J} \]

where \( J \) is the polar moment of inertia of the weld area about the centroid of the weld area
Example: Two weld group

Primary shear

Weld 1 throat area

\[ A_1 = 0.707 \, h_1 \, d_1 = b_1 \, d_1 \]

Weld 2 throat area

\[ A_2 = 0.707 \, h_2 \, d_2 = b_2 \, d_2 \]

Primary shear stress

\[ \tau' = \frac{V}{(A_1 + A_2)} \]

\[ b_1 = 0.707 \, h \]
Two weld group (cont.)

Secondary shear stress

\[ \tau'' = \frac{Mr}{J} \]

where \( J \) is the second polar moment of the area about the centroid, \( G \).
Two weld group (cont.)

Computation of $J$:

Weld area 1

$I_x = \frac{b_1 d_1^3}{12}, \quad I_y = \frac{d_1 b_1^3}{12}$

$J_{G_1} = I_x + I_y = \frac{b_1 d_1^3}{12} + \frac{d_1 b_1^3}{12}$

$J_G = J_{G_1} + A_1 r_1^2$

Weld area 2

$I_x = \frac{b_2 d_2^3}{12}, \quad I_y = \frac{d_2 b_2^3}{12}$

$J_{G_2} = I_x + I_y = \left(\frac{b_2 d_2^3}{12}\right) + \frac{d_2 b_2^3}{12}$

$J_G = J_{G_2} + A_2 r_2^2$

$J = \left(J_{G_1} + A_1 r_1^2\right) + \left(J_{G_2} + A_2 r_2^2\right)$
Unit Second Moment of Area

Note that $J_{G1}$ is proportional to $b_1$ and $J_{G2}$ is proportional to $d_2$

Factoring out the weld widths

$$J_{G1} = b_1 \left( \frac{d_1^3}{12} + d_1 r_1^2 \right) = b_1 J_{u1} = 0.707 h J_{u1}$$

$$J_{G2} = d_2 \left( \frac{b_2^3}{12} + b_2 r_2^2 \right) = b_2 J_{u2} = 0.707 h J_{u2}$$

$$J = 0.707 h \left( J_{u1} + J_{u2} \right)$$

$$J = 0.707 h J_u$$

where $J_{u1}$ and $J_{u2}$ are *unit second moments of the area* which are independent of the weld width.
Table 9-1. Torsional Properties of Fillet Weld Groups

<table>
<thead>
<tr>
<th>Weld</th>
<th>Throat Area</th>
<th>Location of G</th>
<th>Unit Second Polar Moment of Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$A = 0.707 , hd$</td>
<td>$\bar{x} = 0$</td>
<td>$J_u = d^3/12$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\bar{y} = d/2$</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>$A = 1.414 , hd$</td>
<td>$\bar{x} = b/2$</td>
<td>$J_u = \frac{d(3b^2 + d^2)}{6}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\bar{y} = d/2$</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>$A = 0.707h(b + d)$</td>
<td>$\bar{x} = \frac{b^2}{2(b + d)}$</td>
<td>$J_u = \frac{(b + d)^4 - 6b^2d^2}{12(b + d)}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\bar{y} = \frac{d^2}{2(b + d)}$</td>
<td></td>
</tr>
</tbody>
</table>
Table 9-1. Torsional Properties of Fillet Weld Groups (cont.)

| No. | Diagram | Formula for Area | Torsional Inertia 

| 4. | ![Diagram 4](image) | $A = 0.707h(2b + d)$ | $\ddot{x} = \frac{b^2}{2b + d}$ | $J_u = \frac{8b^3 + 6bd^2 + d^3}{12} - \frac{b^4}{2b + d}$ |

| 5. | ![Diagram 5](image) | $A = 1.414h(b + d)$ | $\ddot{x} = \frac{b}{2}$ | $J_u = \frac{(b + d)^3}{6}$ |

| 6. | ![Diagram 6](image) | $A = 1.414 \pi hr$ | $J_u = 2\pi r^3$ |

*G is centroid of weld group; h is weld size; plane of torque couple is in the plane of the paper; all welds are of unit width.*
Example 9-1

A 50-kN load is transferred from a welded fitting into a 200-mm steel channel as illustrated in Fig. 9-14. Estimate the maximum stress in the weld.
Example 9-1 (cont.)

Weld geometry

Primary shear stress (Table 9.1)

\[ A = 0.707(6 \text{ mm})(2 \times 56 \text{ mm} + 190 \text{ mm}) = 1280 \text{ mm}^2 \]

\[ \tau' = \frac{V}{A} = \frac{25 \times 10^3 \text{ N}}{1280 \text{ mm}^2} = 19.5 \text{ MPa} \]

Location of centroid (Table 9.1)

\[ \bar{x} = 0 \]

\[ \bar{y} = \frac{(56 \text{ mm})^2}{2(56 \text{ mm}) + 190 \text{ mm}} = 10.4 \text{ mm} \]
Example 9-1 (cont.)

Moment

\[ M = (25 \times 10^3 \, N) \times (100+10.4) \, \text{mm} \]

\[ = 2760 \, \text{N-m} \]

Second polar moment of the area (Table 9.1)

\[ J = 0.707(6 \, \text{mm}) \left[ \frac{8(56)^3 + 6(56)(190)^2 + (190)^3}{12} - \frac{(56)^4}{2(56)+190} \right] \]

\[ = 7.07 \times 10^6 \, \text{mm}^4 \]
Example 9-1 (cont.)

Secondary shear stresses

$$\tau'' = \frac{Mr}{J}$$

$$r_A = r_B = \left[\frac{(190/2)^2 + (56 - 10.4)^2}{2}\right]^{1/2} = 105 \text{ mm}$$

$$r_C = r_D = \left[\frac{(190/2)^2 + (10.4)^2}{2}\right]^{1/2} = 95.6 \text{ mm}$$

$$\tau_A'' = \frac{Mr''}{J} = \frac{2760(10)^3(105)}{7.07(10)^6} = 41.0 \text{ MPa}$$

$$\tau_C'' = \frac{Mr''}{J} = \frac{2760(10)^3(95.6)}{7.07(10)^6} = 37.3 \text{ MPa}$$
Example 9-1 (cont.)

Resultant shear stress

\[ \tau_A = \tau_B = 37 \text{ MPa} \]

\[ \tau_C = \tau_D = 44 \text{ MPa} \]

Points C and D have highest shear stress
Welded Joints in Bending

(a)

(b) Weld pattern
Welded Joints in Bending – Primary Shear

\[ \tau' = \frac{V}{A} \]

where \( A \) is the total weld throat area
Welded Joints in Bending
Secondary Shear

Bending stress in weld

\[ \sigma = \frac{Mc}{I} \]

To determine \( I \)

\[
I = 2 \left[ \frac{bh^3}{12} + A \left( \frac{d}{2} \right)^2 \right] \approx 2A \left( \frac{d}{2} \right)^2 = \\
= 2(0.707bh) \left( \frac{d}{2} \right)^2 = (0.707h) \left( \frac{bd^2}{2} \right) = \\
= 0.707hI_u
\]
### Table 9-2. Bending Properties of Fillet Welds

<table>
<thead>
<tr>
<th>Weld</th>
<th>Throat Area</th>
<th>Location of $G$</th>
<th>Unit Second Moment of Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$A = 0.707hd$</td>
<td>$\bar{x} = 0$ $\bar{y} = d/2$</td>
<td>$I_u = \frac{d^3}{12}$</td>
</tr>
<tr>
<td>2.</td>
<td>$A = 1.414hd$</td>
<td>$\bar{x} = b/2$ $\bar{y} = d/2$</td>
<td>$I_u = \frac{d^3}{6}$</td>
</tr>
<tr>
<td>3.</td>
<td>$A = 1.414hb$</td>
<td>$\bar{x} = b/2$ $\bar{y} = d/2$</td>
<td>$I_u = \frac{bd^2}{2}$</td>
</tr>
<tr>
<td>4.</td>
<td>$A = 0.707h(2b + d)$</td>
<td>$\bar{x} = \frac{b^2}{2b + d}$ $\bar{y} = d/2$</td>
<td>$I_u = \frac{d^2}{12}(6b + d)$</td>
</tr>
<tr>
<td>5.</td>
<td>[ A = 0.707h(b + 2d) ]</td>
<td>[ \ddot{x} = \frac{b}{2} ]</td>
<td>[ I_u = \frac{2d^3}{3} - 2d^2 \ddot{y} + (b + 2d)\ddot{y}^2 ]</td>
</tr>
<tr>
<td>-----</td>
<td>----------------------------</td>
<td>-----------------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td></td>
<td>[ \ddot{y} = \frac{d^2}{b + 2d} ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>[ A = 1.414h(b + d) ]</td>
<td>[ \ddot{x} = \frac{b}{2} ]</td>
<td>[ I_u = \frac{d^2}{6}(3b + d) ]</td>
</tr>
<tr>
<td></td>
<td>[ \ddot{y} = \frac{d}{2} ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>[ A = 0.707h(b + 2d) ]</td>
<td>[ \ddot{x} = \frac{b}{2} ]</td>
<td>[ I_u = \frac{2d^3}{3} - 2d^2 \ddot{y} + (b + 2d)\ddot{y}^2 ]</td>
</tr>
<tr>
<td></td>
<td>[ \ddot{y} = \frac{d^2}{b + 2d} ]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 9-2. (cont.)

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>8.</td>
<td>$A = 1.414h(b + d)$</td>
<td>$\bar{x} = b/2$</td>
</tr>
<tr>
<td></td>
<td>$\bar{y} = d/2$</td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>$A = 1.414\pi hr$</td>
<td></td>
</tr>
</tbody>
</table>
Welded Joints in Bending Secondary Shear (cont.)

\[ \tau'' = \sigma = \frac{Mc}{I} = \frac{Md}{2} \times \frac{1}{0.707hbd^2 / 2} \]
Example 9-4 – Weld Stresses

Perform an adequacy assessment of the statically loaded welded cantilever carrying 500 lbf depicted in Fig. 9–20. The cantilever is made of AISI 1018 HR steel and welded with a $\frac{3}{8}$-in fillet weld as shown in the figure. An E6010 electrode was used, and the design factor was 3.0.

(a) Use the conventional method for the weld metal.
(b) Use the conventional method for the attachment (cantilever) metal.
(c) Use a welding code for the weld metal.
Example 9-4 – Weld Stresses

Primary Shear

\[ \tau' = \frac{F}{A} = \frac{F}{2(0.707h_1d)} = \frac{500 \text{ lb}}{1.414(0.375 \text{ in})(2 \text{ in})} = 472 \text{ psi} \]

Secondary Shear

\[ \tau'' = \frac{M_r}{I} = \frac{M_r}{(0.707h_1)I_u} = \frac{M_r}{(0.707h_1)(d^3 / 6)} = \frac{(500 \text{ lb})(6 \text{ in})(1 \text{ in})}{0.707(0.375 \text{ in})(2 \text{ in})^3 / 6} = 8500 \text{ psi} \]

\[ \tau = \sqrt{\tau'^2 + \tau''^2} = 8510 \text{ psi} \]
Weld Metal Properties  
(Table 9-3)

<table>
<thead>
<tr>
<th>AWS Electrode Number*</th>
<th>Tensile Strength kpsi (MPa)</th>
<th>Yield Strength, kpsi (MPa)</th>
<th>Percent Elongation</th>
</tr>
</thead>
<tbody>
<tr>
<td>E60xx</td>
<td>62 (427)</td>
<td>50 (345)</td>
<td>17–25</td>
</tr>
<tr>
<td>E70xx</td>
<td>70 (482)</td>
<td>57 (393)</td>
<td>22</td>
</tr>
<tr>
<td>E80xx</td>
<td>80 (551)</td>
<td>67 (462)</td>
<td>19</td>
</tr>
<tr>
<td>E90xx</td>
<td>90 (620)</td>
<td>77 (531)</td>
<td>14–17</td>
</tr>
<tr>
<td>E100xx</td>
<td>100 (689)</td>
<td>87 (600)</td>
<td>13–16</td>
</tr>
<tr>
<td>E120xx</td>
<td>120 (827)</td>
<td>107 (737)</td>
<td>14</td>
</tr>
</tbody>
</table>

Note: Since weld metal strengths are often higher than the base metal, stresses in base metal should be checked.
Permissible Weld Stresses (Table 9-4)

<table>
<thead>
<tr>
<th>Type of Loading</th>
<th>Type of Weld</th>
<th>Permissible Stress</th>
<th>$n^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension</td>
<td>Butt</td>
<td>$0.60S_y$</td>
<td>1.67</td>
</tr>
<tr>
<td>Bearing</td>
<td>Butt</td>
<td>$0.90S_y$</td>
<td>1.11</td>
</tr>
<tr>
<td>Bending</td>
<td>Butt</td>
<td>$0.60$–$0.66S_y$</td>
<td>1.52–1.67</td>
</tr>
<tr>
<td>Simple compression</td>
<td>Butt</td>
<td>$0.60S_y$</td>
<td>1.67</td>
</tr>
<tr>
<td>Shear</td>
<td>Butt or fillet</td>
<td>$0.30S_{ut}$</td>
<td></td>
</tr>
</tbody>
</table>

*The factor of safety $n$ has been computed by using the distortion-energy theory.

†Shear stress on base metal should not exceed $0.40S_y$ of base metal.
Allowable Loads on Fillet Welds
(Table 9-6)

<table>
<thead>
<tr>
<th>Strength Level of Weld Metal (EXX)</th>
<th>60*</th>
<th>70*</th>
<th>80</th>
<th>90*</th>
<th>100</th>
<th>110*</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allowable shear stress on throat, ksi (1000 psi) of fillet weld or partial penetration groove weld</td>
<td>18.0</td>
<td>21.0</td>
<td>24.0</td>
<td>27.0</td>
<td>30.0</td>
<td>33.0</td>
<td>36.0</td>
</tr>
<tr>
<td>( \tau ) =</td>
<td>12.73h</td>
<td>14.85h</td>
<td>16.97h</td>
<td>19.09h</td>
<td>21.21h</td>
<td>23.33h</td>
<td>25.45h</td>
</tr>
<tr>
<td>( \frac{f}{h} ) =</td>
<td>7/8</td>
<td>3/4</td>
<td>5/8</td>
<td>1/2</td>
<td>7/16</td>
<td>3/8</td>
<td>5/16</td>
</tr>
<tr>
<td>Leg Size, in</td>
<td>2.00</td>
<td>1.60</td>
<td>1.25</td>
<td>1.00</td>
<td>0.75</td>
<td>0.63</td>
<td>0.50</td>
</tr>
<tr>
<td>Allowable Unit Force for Various Sizes of Fillet Welds, kip/linear in</td>
<td>10.00</td>
<td>12.50</td>
<td>15.63</td>
<td>20.00</td>
<td>25.00</td>
<td>31.25</td>
<td>40.00</td>
</tr>
<tr>
<td>Material Thickness of Thicker Part Joined, in</td>
<td>To ( \frac{1}{4} ) incl.</td>
<td>Over ( \frac{1}{4} ) To ( \frac{1}{2} )</td>
<td>Over ( \frac{1}{2} ) To ( \frac{3}{4} )</td>
<td>Over ( \frac{3}{4} ) To 1 ( \frac{1}{2} )</td>
<td>Over 1 ( \frac{1}{2} ) To 2 ( \frac{1}{4} )</td>
<td>Over 2 ( \frac{1}{4} ) To 6</td>
<td>Over 6</td>
</tr>
<tr>
<td>Weld Size, in</td>
<td>( \frac{1}{8} )</td>
<td>( \frac{3}{16} )</td>
<td>( \frac{1}{4} )</td>
<td>( \frac{3}{8} )</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{2}{4} )</td>
<td>( \frac{1}{8} )</td>
</tr>
</tbody>
</table>

*Fillet welds actually tested by the joint AISC-AWS Task Committee.
\( f = 0.707h \tau \)

Example 9-4 (cont.)

(a) For E6010, $S_y = 50 \text{ ksi}$

Distortional energy $\Rightarrow S_{sy} = 0.577 \; S_y = 28.9 \text{ ksi}$

Factor of safety, $n = \frac{28.9 \text{ ksi}}{8.51 \text{ ksi}} = 3.39$

(b) For AISI 1018 HR steel, $S_y = 32 \text{ ksi}$

$\sigma = \frac{M c}{I} = \frac{(500 \text{ lb})(6 \text{ in})(1 \text{ in})}{[(.375 \text{ in})(2 \text{ in})^3/12]} = 12 \text{ ksi}$

Factor of safety, $n = \frac{32 \text{ ksi}}{12 \text{ ksi}} = 2.67$
Example 9-4 (cont.)

(c) $\tau = 8.51 \text{ ksi}$, $\tau_{\text{all}} = 18 \text{ ksi}$ (Table 9-6)

Since $\tau < \tau_{\text{all}}$, the weld is adequate

Perform an adequacy assessment of the statically loaded welded cantilever carrying 500 lbf depicted in Fig. 9–20. The cantilever is made of AISI 1018 HR steel and welded with a $\frac{3}{8}$-in fillet weld as shown in the figure. An E6010 electrode was used, and the design factor was 3.0.

(a) Use the conventional method for the weld metal.
(b) Use the conventional method for the attachment (cantilever) metal.
(c) Use a welding code for the weld metal.
Example 9-2

A $\frac{1}{2}$-in by 2-in rectangular-cross-section 1015 bar carries a static load of 16.5 kip. It is welded to a gusset plate with a $\frac{3}{8}$-in fillet weld 2 in long on both sides with an E70XX electrode as depicted in Fig. 9–18. Use the welding code method.

(a) Is the weld metal strength satisfactory?

(b) Is the attachment strength satisfactory?
Example 9-2 (cont.)

(a) From Table 9-6 => 5.57 kip/in

\[ F = 5.57 \text{ (4 in)} = 22.28 \text{ kip} > 16.5 \text{ kip} \Rightarrow \text{OK} \]

(b) From Table A-20, for 1015 steel, \( S_y = 27.5 \text{ ksi} \). From Table 9-4

\[ \tau_{\text{all}} = 0.4 \cdot S_y = 0.4 \cdot (27.5) = 11 \text{ ksi} \]
\[ \sigma_{\text{all}} = 0.6 \cdot S_y = 0.6 \cdot (27.5) = 16.5 \text{ ksi} \]

**Shear stress on base plate**

\[ \tau = \frac{F}{2 \cdot h \cdot l} = \frac{16.5 \text{ lb}}{2 \cdot (0.375 \text{ in}) \cdot (2 \text{ in})} = 11 \text{ ksi} \Rightarrow \text{OK} \]

**Tensile stress in shank**

\[ \sigma = \frac{F}{t \cdot l} = \frac{16.5 \text{ lb}}{(0.5 \text{ in}) \cdot (2 \text{ in})} = 16.5 \text{ ksi} \Rightarrow \text{OK} \]