Chapter 6. Fatigue Failure
Chapter Outline

6–1  Introduction to Fatigue  286
6–2  Chapter Overview  287
6–3  Crack Nucleation and Propagation  288
6–4  Fatigue-Life Methods  294
6–5  The Linear-Elastic Fracture Mechanics Method  295
6–6  The Strain-Life Method  299
6–7  The Stress-Life Method and the S-N Diagram  302
6–8  The Idealized S-N Diagram for Steels  304
6–9  Endurance Limit Modifying Factors  309
6–10  Stress Concentration and Notch Sensitivity  320
6–11  Characterizing Fluctuating Stresses  325
6–12  The Fluctuating-Stress Diagram  327
6–13  Fatigue Failure Criteria  333
6–14  Constant-Life Curves  342
6–15  Fatigue Failure Criterion for Brittle Materials  345
6–16  Combinations of Loading Modes  347
6–17  Cumulative Fatigue Damage  351
6–18  Surface Fatigue Strength  356
6–19  Road Maps and Important Design Equations for the Stress-Life Method  359
We will emphasize topics from the following sections:

6-1 to 6-3  Introduction to fatigue failure and failure theories
6-7        Stress- life method
6-8        S-N diagram
6-9        Endurance limit modifying factors
6-10       Stress concentration and notch sensitivity
6-11 to 13 Fluctuating stresses
Introduction to Fatigue Failure

Fatigue failure of a bolt subjected to repeated unidirectional loads

A – crack initiates at thread root

B – crack propagation (beach marks)

C – final fast fracture
Methods for Predicting Fatigue Failure

1. Strain-Life Method
   - Analyzes localized plastic deformation
   - Accurate for low-cycle fatigue problems

2. Fracture Mechanics Method
   - Predicts incremental crack growth rates

3. Stress-Life Method
   - Traditional method
   - Based on stress levels only
   - Easiest method to implement
   We will consider only the stress-life method
Fatigue Testing

Uniaxial Fatigue Test

Rotating Beam Test
Fatigue Testing Procedure

- Perform test at many different stress levels
- Determine number of cycles to failure
- Plot number of cycles to failure vs. applied stress
- This plot (called the S-N diagram) can be used to estimate the number of cycles to failure for a given stress level
Typical S-N Curve

\( S_{ut} \) – failure at first load cycle

\( S'_e \) – endurance limit (can withstand unlimited load cycles)
Endurance Limit

Endurance limit – the stress level below which failure will not occur, regardless of how many cycles (also known as the fatigue limit)

\[ S_e' \quad \text{- endurance limit under ideal test conditions} \]

\[ S_e \quad \text{- endurance limit under service conditions} \]
Relation between Endurance Limit and Tensile Strength
Relation between Endurance Limit and Tensile Strength (cont.)

\[
S'_e = \begin{cases} 
0.5 \ S_{ut}, & S_{ut} < 200 \text{ ksi} \\
100 \text{ ksi}, & S_{ut} \geq 200 \text{ ksi}
\end{cases}
\]

in SI units

\[
S'_e = \begin{cases} 
0.5 \ S_{ut}, & S_{ut} < 1400 \text{ MPa} \\
700 \text{ MPa}, & S_{ut} \geq 1400 \text{ MPa}
\end{cases}
\]
Endurance Limit Modifying Factors

\[ S_e = k_a k_b k_c k_d k_e k_f S'_e \]

These are called “Marin” factors

- \( k_a \) - Surface condition modifying factor
- \( k_b \) - Size modification factor
- \( k_c \) - Load modification factor
- \( k_d \) - Temperature modification factor
- \( k_e \) - Reliability factor
- \( k_f \) - Miscellaneous effects modification factor
Surface Factor, $k_a$

$$k_a = a S_{ut}^b$$

Table 6–2 Curve Fit Parameters for Surface Factor, Equation (6–18)

<table>
<thead>
<tr>
<th>Surface Finish</th>
<th>Factor $a$</th>
<th>Exponent $b$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_{ut}$, kpsi</td>
<td>$S_{ut}$, MPa</td>
</tr>
<tr>
<td>Ground</td>
<td>1.21</td>
<td>1.38</td>
</tr>
<tr>
<td>Machined or cold-drawn</td>
<td>2.00</td>
<td>3.04</td>
</tr>
<tr>
<td>Hot-rolled</td>
<td>11.0</td>
<td>38.6</td>
</tr>
<tr>
<td>As-forged</td>
<td>12.7</td>
<td>54.9</td>
</tr>
</tbody>
</table>
Size Factor, $k_b$

For bending and torsion

$$k_b = \begin{cases} 
0.879d^{-0.107} & \text{if } 0.11 \leq d \leq 2 \text{ in} \\
0.91d^{-0.157} & \text{if } 2 \leq d \leq 10 \text{ in} \\
1.24d^{-0.107} & \text{if } 2.79 \leq d \leq 51 \text{ mm} \\
1.51d^{-0.157} & \text{if } 51 \leq d \leq 254 \text{ mm}
\end{cases}$$

For axial loads

$$k_b = 1$$
Loading Factor, $k_c$

$$k_c = \begin{cases} 
1 & \text{-- bending} \\
0.85 & \text{-- axial} \\
0.59 & \text{-- torsion}
\end{cases}$$
Temperature Factor, $k_d$

\[ k_d = \frac{S_T}{S_{RT}} \]

\[
\begin{align*}
S_T/S_{RT} & = 0.98 + 3.5 \left(10^{-4}\right) T_F - 6.3 \left(10^{-7}\right) T_F^2 \\
S_T/S_{RT} & = 0.99 + 5.9 \left(10^{-4}\right) T_C - 2.1 \left(10^{-6}\right) T_C^2
\end{align*}
\] (6–26)
Reliability Factor, $k_e$

Accounts for scatter in fatigue data and desired reliability

<table>
<thead>
<tr>
<th>Reliability, %</th>
<th>Transformation Variate $z_a$</th>
<th>Reliability Factor $k_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0</td>
<td>1.000</td>
</tr>
<tr>
<td>90</td>
<td>1.288</td>
<td>0.897</td>
</tr>
<tr>
<td>95</td>
<td>1.645</td>
<td>0.868</td>
</tr>
<tr>
<td>99</td>
<td>2.326</td>
<td>0.814</td>
</tr>
<tr>
<td>99.9</td>
<td>3.091</td>
<td>0.753</td>
</tr>
<tr>
<td>99.99</td>
<td>3.719</td>
<td>0.702</td>
</tr>
</tbody>
</table>
Miscellaneous Effects Factor, $k_f$

Corrosion
Electrolytic plating
Metal spraying
Cyclic frequency
Frettage corrosion
   (corrosion at contact surfaces)
Example 6-6

A 1080 hot-rolled steel bar has been machined to a diameter of 1 in. It is to be placed in reversed axial loading for 70,000 cycles to failure in an operating environment of 650°F. Using ASTM minimum properties, and a reliability for the endurance limit estimate of 99 percent, estimate the endurance limit and fatigue strength at 70,000 cycles.
Example 6-6 - Endurance Limit

Solution
From Table A-20, \( S_{ut} = 112 \) kpsi at 70°F. Since the rotating-beam specimen endurance limit is not known at room temperature, we determine the ultimate strength at the elevated temperature first, using Equation (6-26).

\[
(S_T/S_{RT})_{650^\circ} = 0.98 + 3.5 \times 10^{-4} (650) - 6.3 \times 10^{-7} (650)^2 = 0.94
\]

The ultimate strength at 650°F is then

\[
(S_{ut})_{650^\circ} = (S_T/S_{RT})_{650^\circ} (S_{ut})_{70^\circ} = 0.94 (112) = 105 \text{ kpsi}
\]

The rotating-beam specimen endurance limit at 650°F is then estimated from Equation (6-10) as

\[
S'_{e} = 0.5 (105) = 52.5 \text{ kpsi}
\]

Next, we determine the Marin factors. For the machined surface, Equation (6-18) with Table 6-2 gives

\[
k_a = aS_{ut}^b = 2.0 (105)^{-0.217} = 0.73
\]

For axial loading, from Equation (6-20), the size factor \( k_o = 1 \), and from Equation (6-25) the loading factor is \( k_c = 0.85 \). The temperature factor \( k_d = 1 \), since we accounted for the temperature in modifying the ultimate strength and consequently the endurance limit. For 99 percent reliability, from Table 6-4, \( k_e = 0.814 \). The endurance limit for the part is estimated by Equation (6-17) as

\[
S_o = k_o k_p k_c k_d k_e S'_{e}
\]
\[
= 0.73 (1) (0.85) (1) (0.814) 52.5 = 26.5 \text{ kpsi}
\]

Answer
Example 6-6 - Fatigue Strength at 70,000 cycles

Section 6-8, for $10^3 < N < 10^6$

\[ f = 1.06 - 2.8 \left(10^{-3}\right)S_{ut} + 6.9 \left(10^{-6}\right)S^2_{su} \quad 70 < S_{ut} < 200 \text{ kpsi} \]
\[ f = 1.06 - 4.1 \left(10^{-4}\right)S_{ut} + 1.5 \left(10^{-7}\right)S^2_{su} \quad 500 < S_{ut} < 1400 \text{ MPa} \] (6–11)

\[ S_f = aN^b \] (6–12)

\[ a = \frac{(fS_{ut})^2}{S_e} \] (6–13)

\[ b = -\frac{1}{3} \log \left(\frac{fS_{ut}}{S_e}\right) \] (6–14)
Example 6-6 - Fatigue Strength at 70,000 cycles (cont.)

For the fatigue strength at 70,000 cycles we need to construct the S-N equation. From Equation (6-11), or we could use Figure 6-23.

\[ f = 1.06 - 2.8 \left(10^{-3}\right) \cdot (105) + 6.9 \left(10^{-6}\right)(105)^2 = 0.84 \]

From Equation (6-13),

\[ a = \frac{(f \cdot S_{at})^2}{S_e} = \frac{(0.84 \cdot 105)^2}{26.5} = 293.6 \text{ kpsi} \]

and Equation (6-14)

\[ b = -\frac{1}{3} \log \left(\frac{f \cdot S_{at}}{S_e}\right) = -\frac{1}{3} \log \left[\frac{0.84 \cdot 105}{26.5}\right] = -0.1741 \]

Finally, for the fatigue strength at 70,000 cycles, Equation (6-12) gives

Answer

\[ S_f = a N^b = 293.6 (70,000)^{-0.1741} = 42.1 \text{ kpsi} \]
Stress Concentration and Notch Sensitivity

Static loading (chapter 3) \( \Rightarrow \) \( K_t \) and \( K_{ts} \)

Fatigue loading – some materials are less sensitive to stress concentrations \( \Rightarrow \) reduced stress concentration

Fatigue stress concentration factors

\[
\sigma_{\text{max}} = K_f \sigma_o \\
\tau_{\text{max}} = K_{fs} \tau_o
\]
Notch Sensitivity

\[ q = \frac{K_f - 1}{K_t - 1} \]

\[ q_{\text{shear}} = \frac{K_{fs} - 1}{K_{ts} - 1} \]

where \( 0 \leq q, q_{\text{shear}} \leq 1 \)

Note:

\( q=0 \Rightarrow K_f = 1 \) (no sensitivity to notches)

\( q=1 \Rightarrow K_f = K_t \) (no reduction due to fatigue)
Notch Sensitivity – Axial and Bending Loading

**Figure 6–26** Notch-sensitivity charts for steels and UNS A92024-T wrought aluminum alloys subjected to reversed bending or reversed axial loads. For larger notch radii, use the values of $q$ corresponding to the $r = 0.16$-in (4-mm) ordinate.

Neuber Equation
(basis for Fig. 6-26 - steels)

\[ K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a}/\sqrt{r}} \]  \hspace{2cm} (6–34)

Bending or axial:

\[ \sqrt{a} = 0.246 - 3.08 \times 10^{-3} S_{ut} + 1.51 \times 10^{-5} S_{ut}^2 - 2.67 \times 10^{-8} S_{ut}^3 \quad 50 \leq S_{ut} \leq 250 \text{ kpsi} \]  \hspace{2cm} (6–35)

\[ \sqrt{a} = 1.24 - 2.25 \times 10^{-3} S_{ut} + 1.60 \times 10^{-6} S_{ut}^2 - 4.11 \times 10^{-10} S_{ut}^3 \quad 340 \leq S_{ut} \leq 1700 \text{ MPa} \]

Torsion:

\[ \sqrt{a} = 0.190 - 2.51 \times 10^{-3} S_{ut} + 1.35 \times 10^{-5} S_{ut}^2 - 2.67 \times 10^{-8} S_{ut}^3 \quad 50 \leq S_{ut} \leq 220 \text{ kpsi} \]  \hspace{2cm} (6–36)

\[ \sqrt{a} = 0.958 - 1.83 \times 10^{-3} S_{ut} + 1.43 \times 10^{-6} S_{ut}^2 - 4.11 \times 10^{-10} S_{ut}^3 \quad 340 \leq S_{ut} \leq 1500 \text{ MPa} \]
Notch Sensitivity – Torsion Loading

Figure 6–27  Notch-sensitivity curves for materials in reversed torsion. For larger notch radii, use the values of $q_s$ corresponding to $r = 0.16$ in (4 mm).
Solving for $K_f$ and $K_{fs}$ gives

$$K_f = 1 + q(K_t - 1)$$

$$K_{fs} = 1 + q_{shear}(K_{ts} - 1)$$
Example 6-7

A steel shaft in bending has an ultimate strength of 690 MPa and a shoulder with a fillet radius of 3 mm connecting a 32-mm diameter with a 38-mm diameter. Estimate $K_f$ using:

(a) Figure 6-26.

(b) Equations (6-34) and (6-35).

Solution

From Figure A-15-9, using $D/d = 38/32 = 1.1875$, $r/d = 3/32 = 0.09375$, we read the graph to find $K_t = 1.65$.

(a) From Figure 6-26, for $S_{ut} = 690$ MPa and $r = 3$ mm, $q = 0.84$. Thus, from Equation (6-32)

$$K_f = 1 + q (K_t - 1) = 1 + 0.84 (1.65 - 1) = 1.55$$

(b) From Equation (6-35) with $S_{ut} = 690$ MPa, $\sqrt{a} = 0.314 \sqrt{\text{mm}}$. Substituting this into Equation (6-34) with $r = 3$ mm gives

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}} = 1 + \frac{1.65 - 1}{1 + \frac{0.314}{\sqrt{3}}} = 1.55$$

Answer
Example 6-7 (cont.)

Figure A–15–9  Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$. 

$K_I$  

$D/d = 3$  

$D/d = 1.5$  

$D/d = 1.10$  

$D/d = 1.05$  

$D/d = 1.02$  

$r/d$  

$M$  

$M$  

$D$  

$d$
Characterization of Fluctuating Stress

Zero Mean Stress (ex. Rotating beam experiment)

\[ \sigma_m = 0 \]

- \( \sigma_m \) – mean stress (\( = 0 \))
- \( \sigma_a \) – stress amplitude
- \( \sigma_r \) – stress range
Characterization of Fluctuating Stress (cont.)

Non-zero mean stress (shafts under combined loading)

\[\sigma_m \quad \text{– mean stress}\]
\[\sigma_a \quad \text{– stress amplitude}\]
\[\sigma_{\text{max}} \quad \text{– maximum stress}\]
\[\sigma_{\text{min}} \quad \text{– minimum stress}\]
\[\sigma_r \quad \text{– stress range}\]
Fluctuating Stress (cont.)

![Graph showing fluctuating stress](image)

**Figure 6-33** Plot of fatigue failures for mean stresses in both tensile and compressive regions. Normalizing the data by using the ratio of steady strength component to tensile strength $\sigma_m / S_{ut}$, steady strength component to compressive strength $\sigma_m / S_{uc}$ and strength amplitude component to endurance limit $\sigma_a / S_e$ enables a plot of experimental results for a variety of steels.
Fatigue Failure Criteria for Non-zero Mean Stress

Figure 6-36  Comparison of several infinite life fatigue failure criteria.
Yield Line (Langer Line)

Yielding occurs if

\[ \sigma_a + |\sigma_m| = S_y \]

Factor of safety associated with yielding

\[ n_y = \frac{S_y}{\sigma_a + |\sigma_m|} \]
Goodman Failure Criteria

Failure criterion:
\[
\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = 1 \quad (6-40)
\]

Design equation:
\[
N_f = \left( \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} \right)^{-1} \sigma_m \geq 0 \quad (6-41)
\]

See text for other failure criteria (Gerber, ASME elliptic, etc.)
EXAMPLE 6–9

A steel bar undergoes cyclic loading such that at the critical notch location the nominal stress cycles between $\sigma_{\text{max}} = 40 \text{kpsi}$ and $\sigma_{\text{min}} = 20 \text{kpsi}$, and a fatigue stress-concentration factor is applicable with $K_f = 1.2$. For the material, $S_{\text{mt}} = 100 \text{kpsi}$, $S_y = 85 \text{kpsi}$, and a fully corrected endurance limit of $S_e = 40 \text{kpsi}$. Estimate $(a)$ the fatigue factor of safety based on achieving infinite life according to the Goodman line. $(b)$ the yielding factor of safety.

Solution

(a) From Equations (6–8) and (6–9),

$$\sigma_{d0} = \frac{40 - 20}{2} = 10 \text{kpsi} \quad \sigma_{m0} = \frac{40 + 20}{2} = 30 \text{kpsi}$$

Applying Equations (6–38) and (6–39),

$$\sigma_a = K_f \sigma_{d0} = 1.2 \times 10 = 12 \text{kpsi}$$
$$\sigma_m = K_f \sigma_{m0} = 1.2 \times 30 = 36 \text{kpsi}$$

For a positive mean stress, apply Equation (6–41),

Answer

$$n_f = \left( \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{\text{mt}}} \right)^{-1} = \left( \frac{12}{40} + \frac{36}{100} \right)^{-1} = 1.52$$
Example 6-9 (cont.)

(b) To avoid even localized yielding at the notch, keep $K_f$ applied to the stresses for the yield check. Using Equation (6-43),

$$n_y = \frac{\delta_y}{\sigma_a + |\sigma_m|} = \frac{85}{12 + 36} = 1.8$$

No yielding is predicted at the notch at the first stress cycle. Of course, realize that with continued cycling, at the grain level the cyclic stress will eventually lead to very localized plastic strain (see Section 6-3). If there were truly no plastic strain, there would be no fatigue.