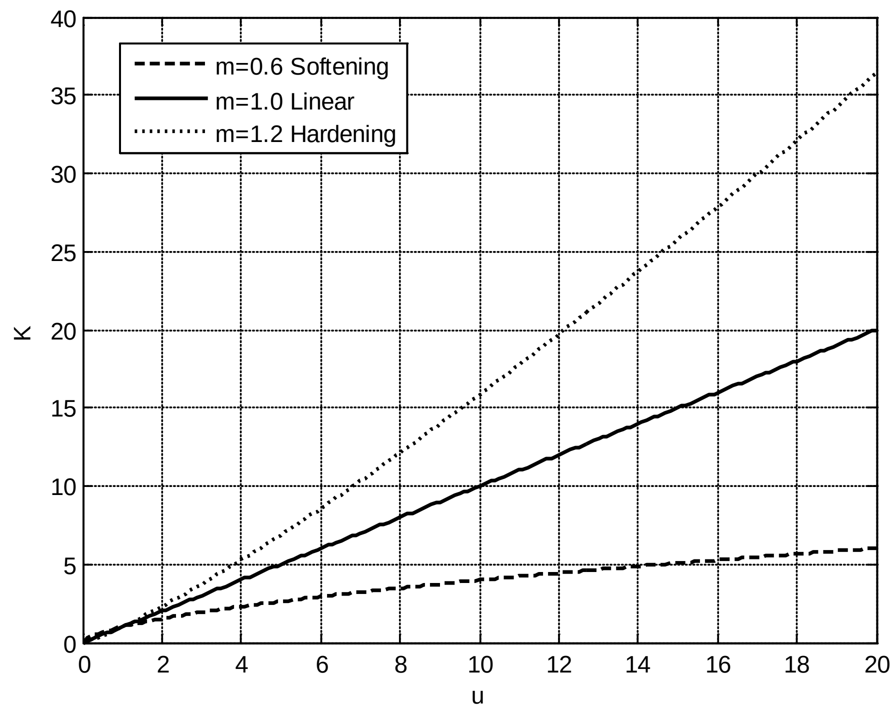
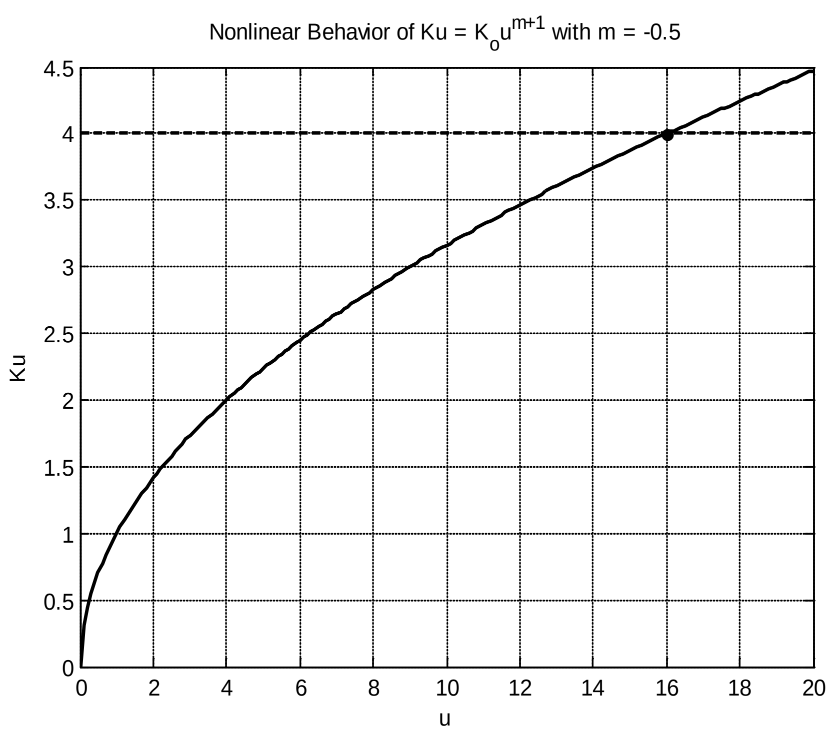
One Degree of Freedom Nonlinear Example

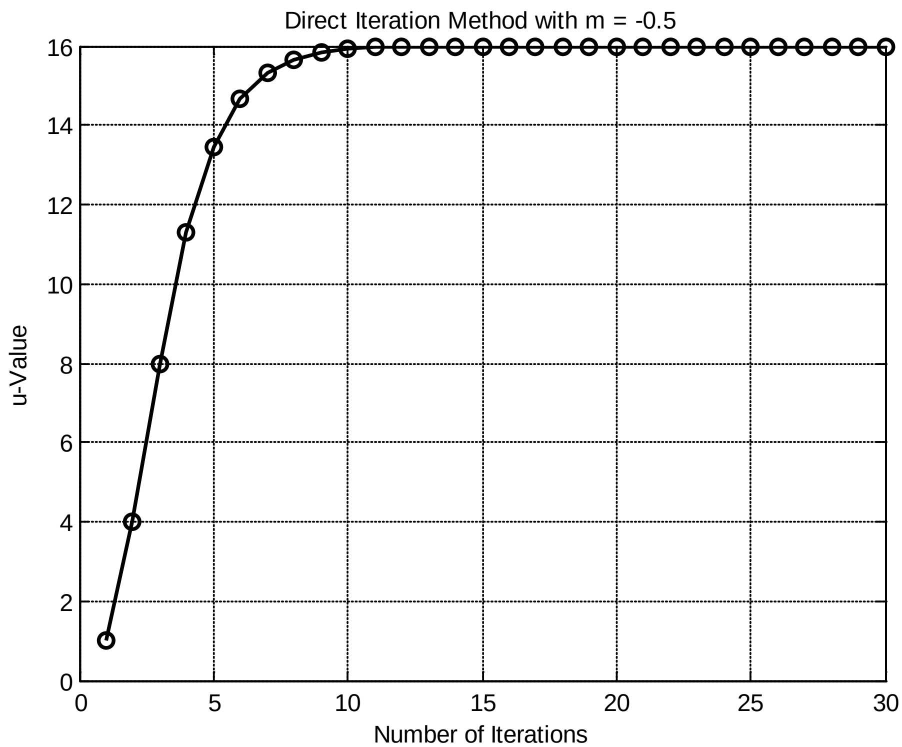
Consider the basic finite element equation $[K]\{u\} = \{F\}$ for the one degree of freedom case with a specific nonlinear stiffness $K = K(u) = K_o u^m$. Particular examples of this stiffness response for different values of *m* are shown in the following figure.

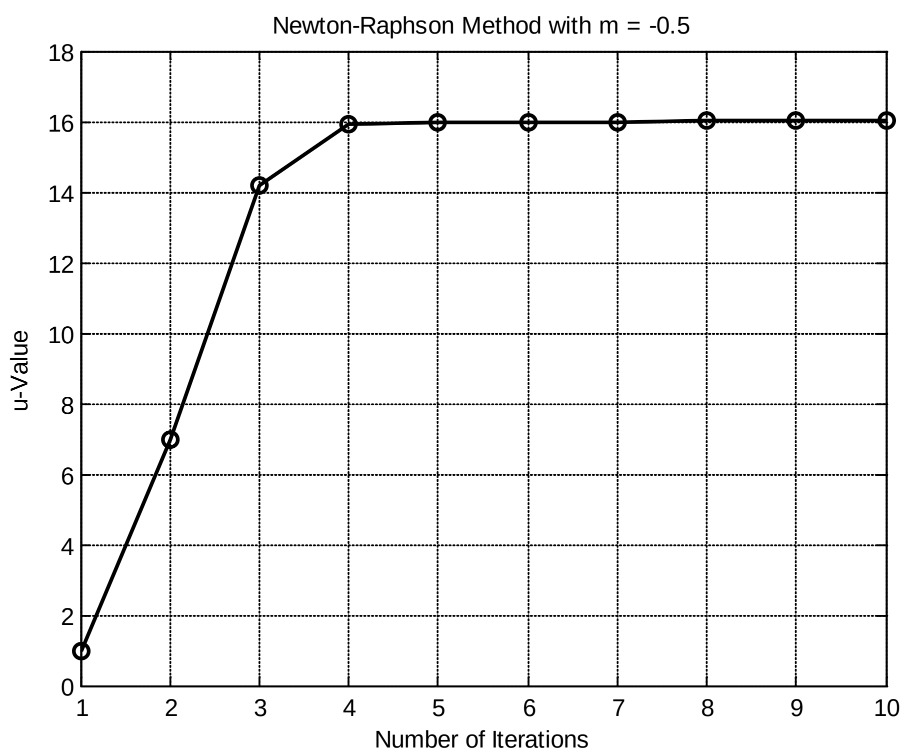


We now consider two examples, including both convex and concave behaviors for $[K(u)]\{u\}$. For the convex case, we choose *m* = -0.5 and take *F* = 4 as shown in the figure below. The highlighted dot indicates where $[K]\{u\} = \{F\}$.

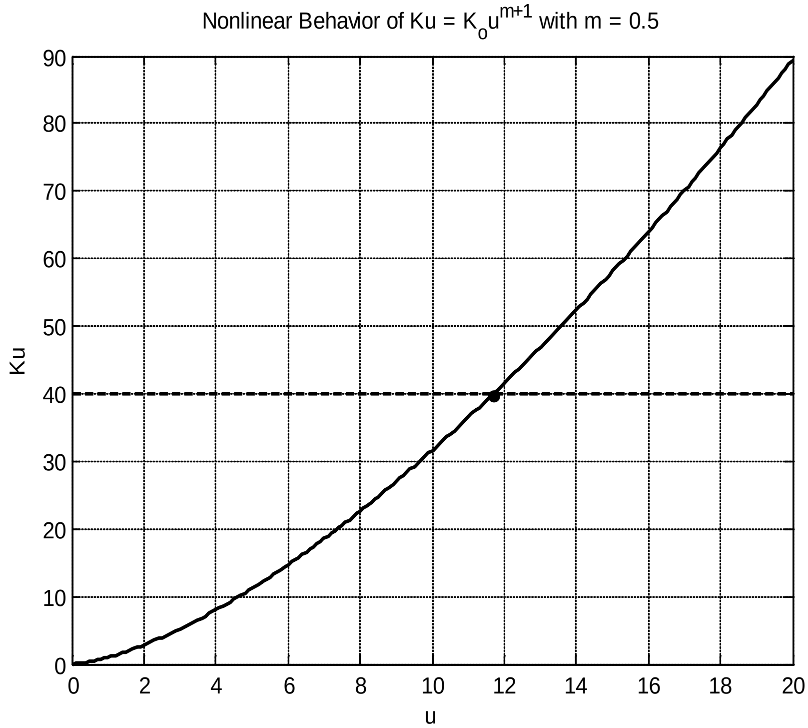


In order to find the solution for this case, we employ both the Direct Iteration and Newton-Raphson methods. The results are illustrated in the two figures below for the case with $u^o = 1$. It should be noted that both methods converge to the same answer $u \approx 16$, but the number of iterations necessary to find the solution is much smaller using the Newton_Raphson scheme.

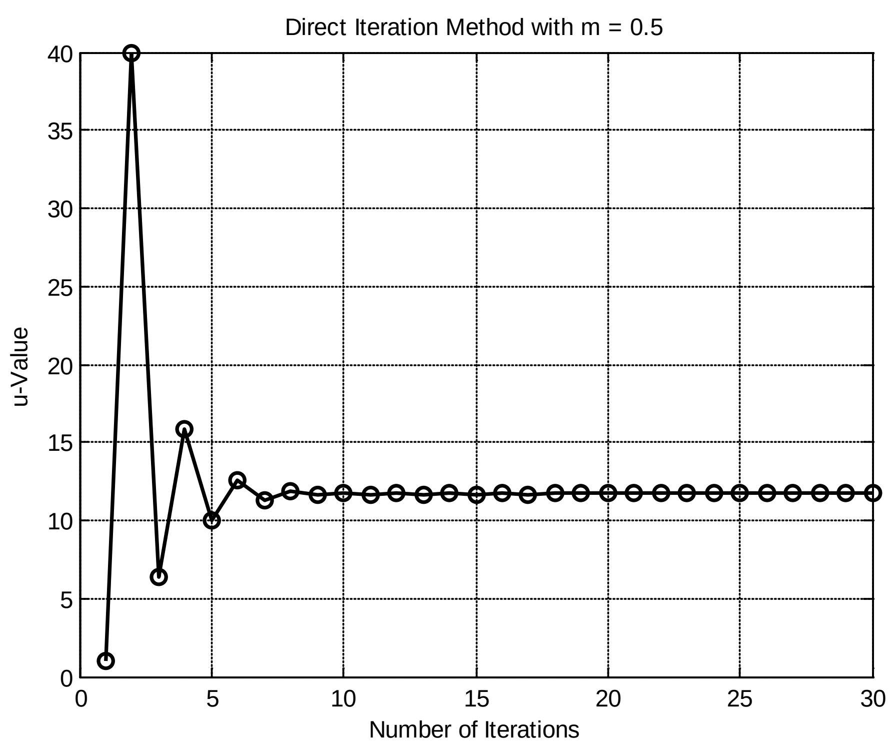


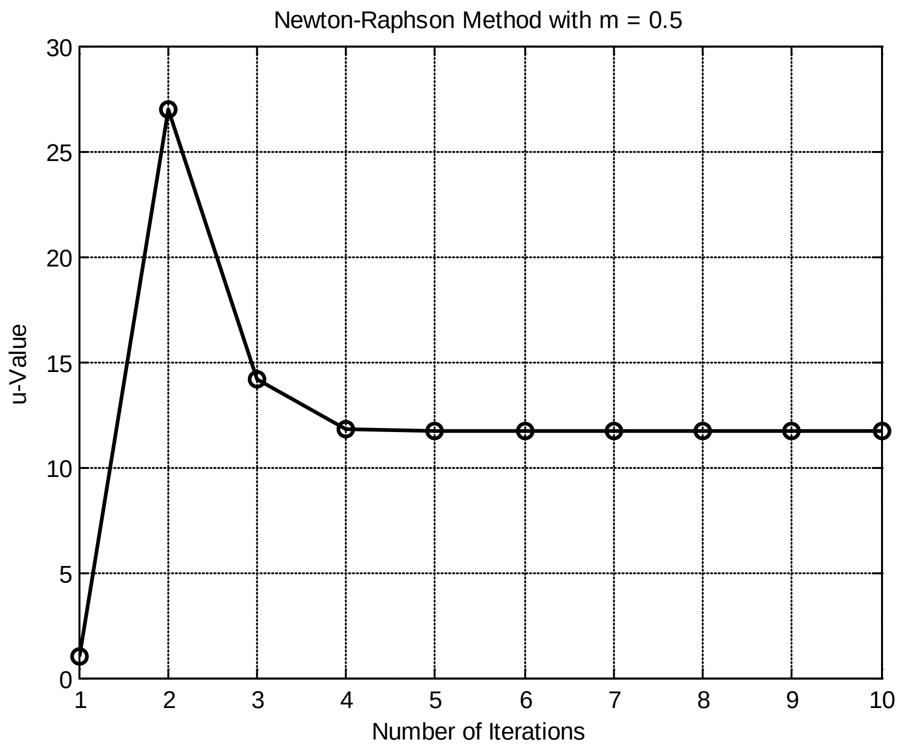


Finally consider the concave case with m = 0.5 and take F = 40 as shown in the figure below. Again, the highlighted dot indicates where $[K]\{u\} = \{F\}$.



We again employ both the Direct Iteration and Newton-Raphson methods to find the solution, and the results are illustrated in the following two figures for the case with $u^o = 1$. It should be noted that both methods converge to the same answer $u \approx 11.7$, but the number of iterations necessary to find the solution is again much smaller using the Newton_Raphson scheme.

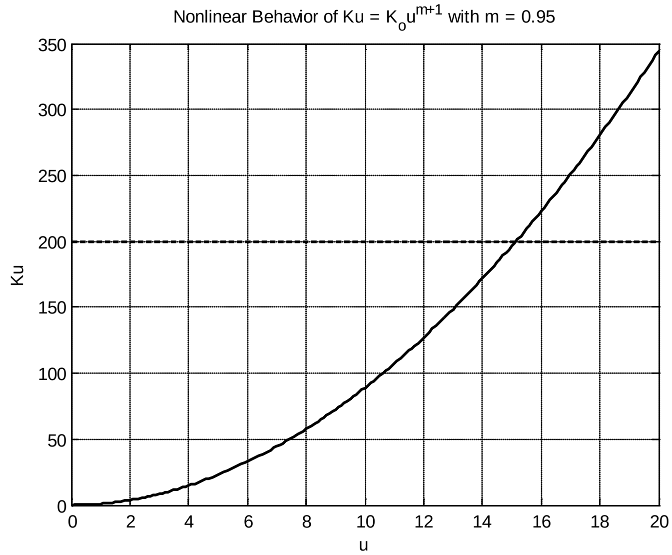


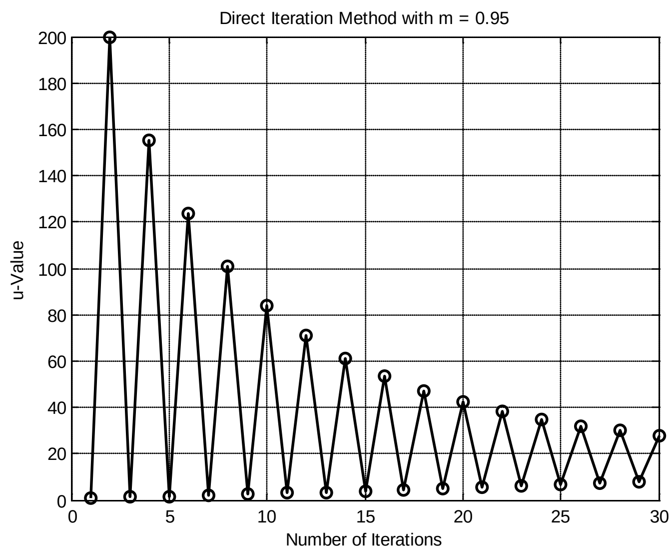


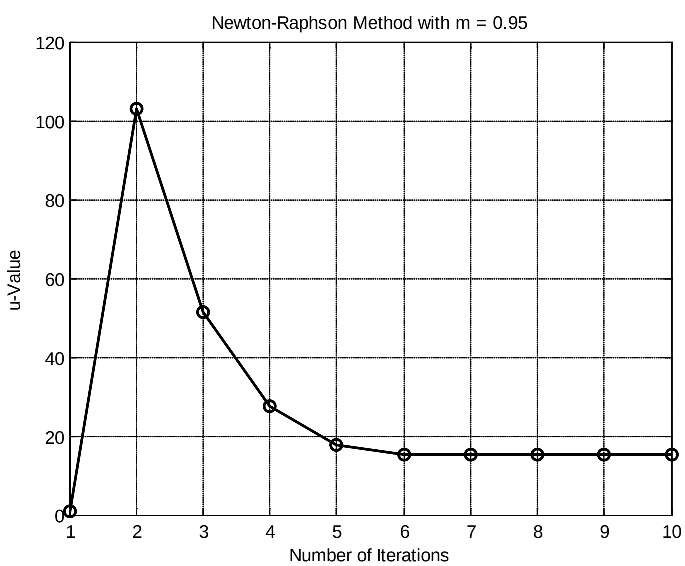
The MATLAB computer code used to calculate and plot these results is given below.

```
% MCE 561 - Example Nonlinear Solution  Ku=F with
K=Ko*u^m
clc;clear all;clf;K0=1;
% Plot General Functional Behavior of K vs u for Different m-
Values
for m=[0.6,1,1.2]
u=0:0.1:20;
K=K0*u.^m;
figure(1)
plot(u,K,'k','linewidth',2)
hold on
xlabel('u');ylabel('K');grid on
legend('m=0.6','m=1.0','m=1.2')
end
% Plot Functional Behavior of Ku vs u
m=0.5;F=40;
U=0:0.1:20;
KU=K0*U.^(m+1);
figure(2)
plot(U,KU,'k','linewidth',2)
hold on
plot(U,F*ones(1,length(U)),'--k','linewidth',2)
xlabel('u');ylabel('Ku');grid on
title(['Nonlinear Behavior of Ku = K_ou^m^+^1 with m =
',num2str(m)])
% Direct Iteration Routine
un=1;
for n=1:30
   iter(n)=n;
   K=K0*un^m;
   un1=(K^-1)*F;
   ud(n)=un;
   un=un1;
end
% Plot Iteration Results
figure(3)
plot(iter,ud,'k-o','linewidth',2)
xlabel('Number of Iterations')
ylabel('u-Value')
title(['Direct Iteration Method with m = ',num2str(m)])
grid on
% Newton-Raphson Method
unr=1;
for n=1:10
   it(n)=n;
   K=K0*unr^m;
   K1=m*K0*unr^(m-1);
   R=K*unr-F;
   J=K+K1*unr;
   dunr=-(J^-1)*R;
   Unr(n)=unr;
   unr=unr+dunr;
end
figure(4)
plot(it,Unr,'k-o','linewidth',2)
xlabel('Number of Iterations')
ylabel('u-Value')
title(['Newton-Raphson Method with m = ',num2str(m)])
grid on
```

Example with *m* = 0.95







Example with *m* = 1.0