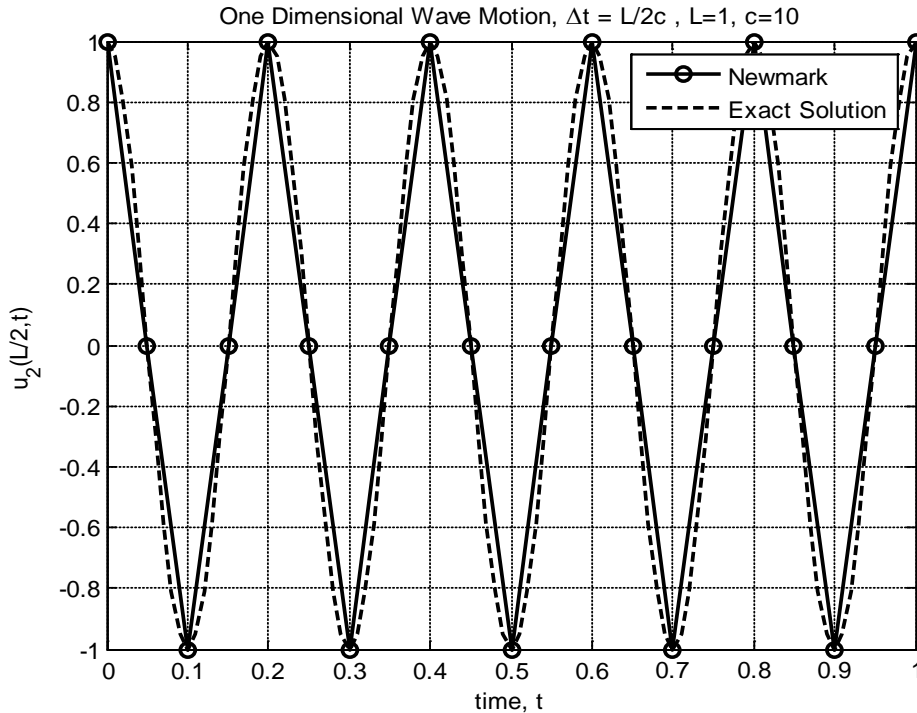


Extended Solution to Homework Problem 9-2

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}, \quad c = \sqrt{E/\rho} \quad u(0,t) = u(L,t) = 0, \quad u(x,0) = \sin\left(\frac{\pi x}{L}\right), \quad \dot{u}(x,0) = 0$$

$$\text{Exact Solution : } u(x,t) = \sin\left(\frac{\pi x}{L}\right) \cos\left(\frac{\pi ct}{L}\right)$$

$$\text{Newmark FEA Solution : } (u_2)_{n+1} = u(L/2, t_{n+1}) = \frac{2}{3}(u_2)_n + \frac{L}{3c}(\dot{u}_2)_n + \frac{L^2}{18c^2}(\ddot{u}_2)_n$$



```
% MCE 561 Spring 2011 Prof. Sadd
% Extended Solution to Problem 9-2
clc;clear all;clf
u=[];ut=[];utt=[];
c=10;L=1;dt=L/(2*c);
u(1)=1;ut(1)=0;utt(1)=-12*(c^2)/(L^2);
for n=1:20
u(n+1)=(2/3)*u(n)+(L/(3*c))*ut(n)+((L^2)/(18*c^2))*utt(n);
utt(n+1)=-12*(c^2)/(L^2)*u(n+1);
ut(n+1)=ut(n)+(0.5*utt(n)+0.5*utt(n+1))*dt;
end
n=[1:21];t=dt*(n-1);
plot(t,u,'ko-','linewidth',2)
xlabel('time, t');ylabel('u_2(L/2,t)')
title('One Dimensional Wave Motion, \Deltat = L/2c')
hold on; grid on;
te=0:0.01:1;
ue=cos(pi*c*te/L);
plot(te,ue,'k--','linewidth',2)
legend('Newmark','Exact Solution')
```