Consider the third order Euler-Cauchy ordinary differential equation example that was solved by hand in Example 4, p112 in the text. The problem is stated as

$$x^3 y''' - 3x^2 y'' + 6xy' - 6y = 0$$

The problem had the initial conditions $y(1) = 2$, $y'(1) = 1$, $y''(1) = -4$, which produced the following analytical solution

$$y = 2x + x^2 - x^3$$

We now wish to use MATLAB (ode45) to find the numerical solution to this problem and compare with result (2). Following the usual scheme we want to express the single 3rd order ODE (1) as a system of three first order ODE's. The procedures are similar to our previous second order example. So we first solve for the highest order derivative in the original equation (1)

$$y''' = \frac{1}{x^3} \left(3x^2 y'' - 6xy' + 6y\right)$$

Next introduce the following three new system variables

$$y_1 = y, \quad y_2 = y', \quad y_3 = y''$$

and thus we can re-write the original 3rd order ODE as

$$\frac{dy_1}{dx} = y_2$$
$$\frac{dy_2}{dx} = y_3$$
$$\frac{dy_3}{dx} = \frac{1}{x^3} \left(3x^2 y_3 - 6xy_2 + 6y_1\right)$$

or as a matrix system

$$\begin{bmatrix} y_1' \\ y_2' \\ y_3' \end{bmatrix} = \begin{bmatrix} y_2 \\ y_3 \\ \frac{1}{x^3} \left(3x^2 y_3 - 6xy_2 + 6y_1\right) \end{bmatrix}$$

This can then be numerically solved using MATLAB following the same procedures and coding done in our previous examples. The following code handles this particular problem and the figure illustrates the comparison with the exact analytical solution (2)
Numerical Solution of $x^3y'''+3x^2y''+6xy'-6y=0$

% MCE 372 Engineering Analysis Example Code - Prof. M. Sadd
% Solution of Third Order Euler-Cauchy ODE
% Example 4, p112 Text
% Comparison with Exact Analytical Solution

function main
clc;clear all;clf
% Enter initial condition matrix
yo=[2,1,-4];
[x,y]=ode45(@DE3,[1:0.1:5],yo);
plot(x,y(:,1),'k-','Linewidth',1.5)
xlabel('x'),ylabel('y'),grid on
title('Numerical Solution of $x^3y'''+3x^2y''+6xy''-6y=0$')
hold on
% Plot Exact Solution
X=1:0.5:5;Y=2*X+X.^2-X.^3;
plot(X,Y,'or','markerfacecolor','r')
legend('Numerical Solution','Exact Solution')
function dydx=DE3(x,y)
% Computes Derivatives of Each Equation
dydx=[y(2);y(3);(3*x.^2.*y(3)-6*x.*y(2)+6*y(1))./(x.^3)];