Numerical Integration Using MATLAB
Applications in Vector Integral Calculus

MATLAB can numerically evaluate single, double and triple integrals found in engineering applications. In regard to triple integrals, the MATLAB command

\[ \text{triplequad}(\text{fun}, \text{xmin}, \text{xmax}, \text{ymin}, \text{ymax}, \text{zmin}, \text{zmax}) \]

evaluates the triple integral of the function \( \text{“fun}(x,y,z)” \) over a three dimensional rectangular region: \( \text{xmin} \leq x \leq \text{xmax}, \text{ymin} \leq y \leq \text{ymax}, \text{zmin} \leq z \leq \text{zmax} \). A simple example of using this command would be to compute the total mass \( M \) of a nonhomogeneous rectangular solid with a given mass density \( \rho = \rho(x,y,z) \) that depends on spatial coordinates. The relation for the total mass is then given by

\[ M = \iiint_V \rho(x,y,z) dV \]

Exercise 10.7.1 from the Kreyszig text provides a specific case with
\( \rho = x^2 + y^2 + z^2 \) over the region \(-4 \leq x \leq 4, -1 \leq y \leq 1, 0 \leq z \leq 2\), and so the relation for this case could be expressed as

\[ M = \int_{-4}^{4} \int_{-1}^{1} \int_{0}^{2} (x^2 + y^2 + z^2) dx dy dz \]

The MATLAB code that calculates this expression is given in the text box below along with the output from the command window.

```matlab
% MCE 372 Numerical Integration Example
Using "triplequad"
clear all
F=@(x,y,z)x.^2+y.^2+z.^2;
Mtotal = triplequad(F,-4,4,-1,1,0,2)
```

\[ M_{\text{total}} = 224 \]

Note the use of the anonymous function \( \text{F=@}(x,y,z)x.^2+y.^2+z.^2 \) to properly provide the integrand for the numerical integration command.

We next consider an application within vector integral calculus connected with the Divergence Theorem
\[ \iiint_{V} \nabla \cdot \mathbf{F} \, dV = \iint_{S} \mathbf{F} \cdot \mathbf{n} \, dS \]

which provides a relation between double and triple integrals. This relation can be used to evaluate the surface integral (which is sometimes more complicated) by evaluating the volume integral. In the current scheme, we evaluate the triple integral using MATLAB.

Consider the specific example with \( \mathbf{F} = xy^2 \mathbf{i} + xy \mathbf{j} + yz^2 \mathbf{k} \) over the box \( |x| \leq 1, |y| \leq 4, 0 \leq z \leq 4 \). For this case \( \nabla \cdot \mathbf{F} = y^2 + x + 2yz \). The MATLAB code that calculates the volume integral is given in the text box below along with the output from the command window.

```matlab
% MCE 372 Numerical Integration Examples Using "triplequad"
clear all
% Using Divergence Thm. Calculate Surface Integral
F=@(x,y,z) y.^2+x+2*y.*z;
I=triplequad(F,-1,1,-4,4,0,4)

I =
341.3333
>>
```

Also recall from previous discussion on potential theory and harmonic functions that we could use the Divergence Theorem to develop the expression

\[ \iiint_{V} \nabla^2 f \, dV = \iint_{s} \hat{\mathbf{n}} \cdot \nabla f \, dS \]

This relation can then be used to evaluate the surface integral of the normal derivative (often related to a net flux out) by evaluating instead the volume integral using MATLAB. Consider the specific example where \( f = x^2 y + y^2 x + z^2 \Rightarrow \nabla^2 f = 2y + 2x + 2 \) over the unit cube \( 0 \leq (x, y, z) \leq 1 \). The MATLAB code that calculates the volume integral is given in the text box below along with the output from the command window.

```matlab
% MCE 372 Numerical Integration Examples Using "triplequad"
clear all
% Potential Theory/Harmonic Function Example
% Use Divergence Thm. to Calculate Surface Integral of Normal Derivative
f=@(x,y,z) 2*y+2*x+2;
I=triplequad(f,0,1,0,1,0,1)

I =
4
>>
```