MATLAB Solution of Higher Order Differential Equations

To use MATLAB ODE solvers for equations of orders higher than 1, we must first write the equation as a system of first order equations. This is easily done as demonstrated in the following example.

Second Order Example

Consider the following well-known 2nd order ODE with constant coefficients *a,b,c* and forcing function *f*(*t*)

$$a\frac{d^2y}{dt^2}+b\frac{dy}{dt}+cy=f(t)$$

The general approach is to solve for the highest derivative

$$\frac{d^2y}{dt^2}=\frac{1}{a}\left(f(t)-b\frac{dy}{dt}-cy\right)$$

Next define two new variables $x_1=y$ and $x_2=\dot{y}$ and so this allows us to write the original ODE as the two equations

$$\frac{dx_1}{dt}=x_2$$

$$\frac{dx_2}{dt}=\frac{1}{a}\left(f(t)-bx_2-cx_1\right)$$

or in matrix form

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}x_2\\ \frac{1}{a}\left(f(t)-bx_2-cx_1\right)\end{bmatrix}$$

This is now a first order matrix ODE and the form is sometimes called the *Cauchy* or *state variable* form. Note that this same scheme can be used for ODEs with orders higher than two.

To use any of the MATLAB ODE solvers we simply follow the previous coding but apply it to the array of ODEs.

Consider the specific case with *a* = *c* = 1, *b* = *f*(*t*) = 0. This yields the equation

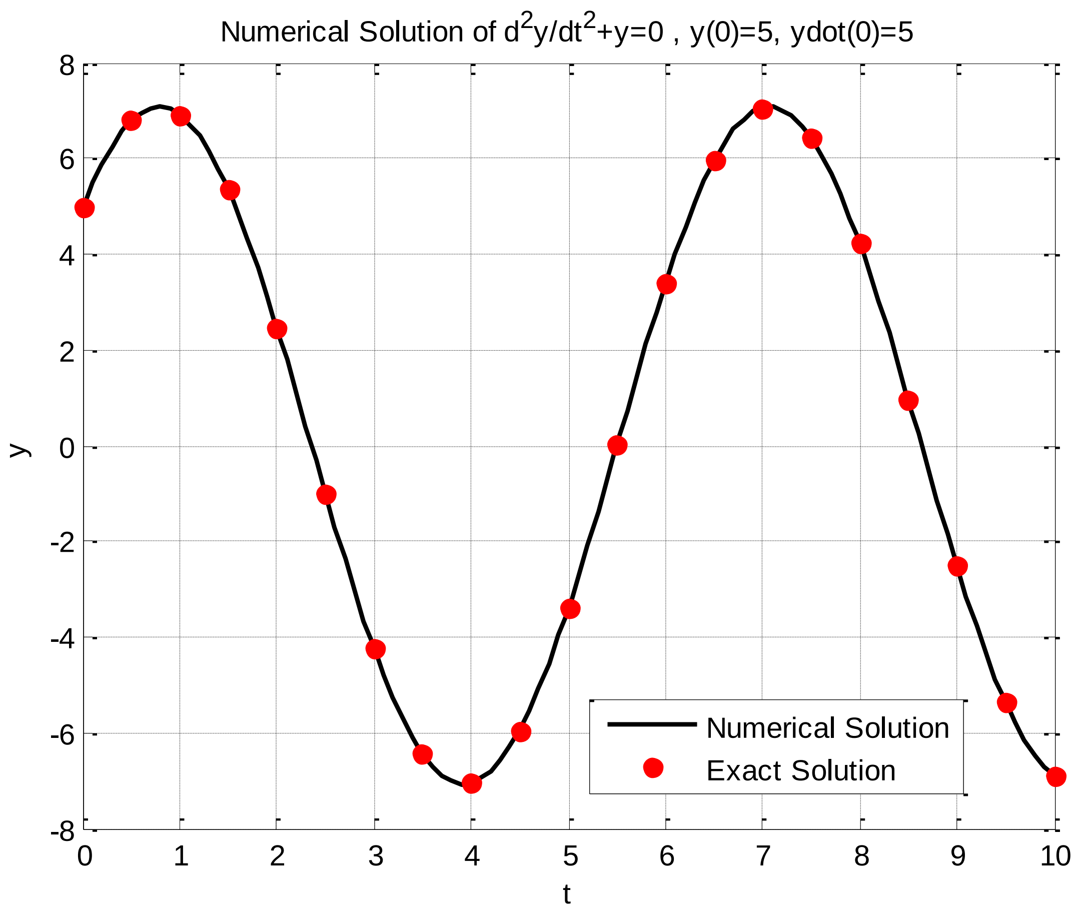
$$\frac{d^2y}{dt^2} + y = 0$$

that we have seen before. Recall that the exact solution was given by $y = A\sin t + B\cos t$, where the constants are $A = x_2(0),\ B = x_1(0)$. We will choose the specific initial conditions $x_1(0) = y(0) = 5$ and $x_2(0) = \dot{y}(0) = 5$

Following the usual coding from the first order examples, the MATLAB code that will solve this second order example is listed below. The code simply handles the problem in a matrix format and plots both the numerical and exact analytical solutions.

```
% MCE 372 Engineering Analysis Example Code
% Solution of Second Order ODE d2y/dt2+y = 0
% Exact Analytical Solution: y=xo(2)*sint+xo(1)*cost
function main
clc;clear all;clf
% Enter initial condition matrix
xo=[5,5];
[t,x]=ode45(@DE2,[0:0.1:10],xo);
plot(t,x(:,1),'k-','Linewidth',1.5)
xlabel('t'),ylabel('y'),grid on
title('Numerical Solution of d^2y/dt^2+y=0 , y(0)=5, dot(0)=5')
hold on
% Plot Exact Solution
T=0:0.5:10;Y=xo(2)*sin(T)+xo(1)*cos(T);
plot(T,Y,'or','markerfacecolor','r')
legend('Numerical Solution','Exact Solution')
function dxdt=DE2(t,x)
% Computes Derivatives of Each Equation
dxdt=[x(2);-x(1)];
```

The solution plots are given by


Numerical Solution of d2y/dt2+y=0 , y(0)=5, ydot(0)=5
y
t
8
6
4
2
0
-2
-4
-6
-8
0
1
2
3
4
5
6
7
8
9
10
Numerical Solution
Exact Solution