Fourier Series Example – MATLAB Evaluation

Square Wave Example

Consider the following square wave function defined by the relation

\[
f(x) = \begin{cases} 
1, & 0 < x < 0.5 \\
-1, & 0.5 < x < 1 
\end{cases}
\]

This function is shown below.

We will assume it has an odd periodic extension and thus is representable by a Fourier Sine series

\[
f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}, \quad b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin \frac{n\pi x}{L} dx, \quad n = 1, 2, 3, \ldots
\]

where \( L = 1/2 \). Evaluating the Fourier coefficients gives

\[
b_n = 2 \int_{-1/2}^{1/2} f(x) \sin 2n\pi x dx = 2 \int_{0}^{0} (-1) \sin 2n\pi x dx - 2 \int_{0}^{1/2} (1) \sin 2n\pi x dx = \frac{2}{n\pi} [1 - (-1)^n]
\]

and thus the series representation reduces to

\[
f(x) = \sum_{n=1, 3, 5, \ldots}^{\infty} \frac{4}{n\pi} \sin 2n\pi x
\]
The evaluation of this series can easily be done using MATLAB and a code is given in the text box below. This code loops and evaluates the partial sums of the series and plots them until an error criteria is satisfied. A couple of the plots are shown below.

```matlab
% MCE 372 Engineering Analysis Example Code - Prof. Sadd
% Fourier Series Evaluation & Plotting Example
% Square Wave Function
% Sum & Plot FS Partial Sums Until Error Level Is Reached
% ***********************************************
clc;clear all;clf;
x=linspace(0,1.5,1000);
f=0;
for n=1:2:200
    % First Plot Square Wave Function
    X=[0,0,0.5,0.5,1.0,1.0,1.5,1.5];
    Y=[0,1,1,1,-1,-1,1,1,0];
    line(X,Y,'color','r','linewidth',2)
    grid on;hold on;
    f=f+(4/(n*pi))*sin(2*pi*n*x);
    error=mean((abs(f)-1).^2);
    plot(x,f,'k','linewidth',2)
    title(['Square Wave FS Partial Sum:   n = ',num2str(n),'
          Error = ',num2str(error)])
    pause
    if error<0.01
        break
    end
    clf
end
```

![Square Wave FS Partial Sum: n = 5  Error = 0.067914](image1.png)

![Square Wave FS Partial Sum: n = 21  Error = 0.019523](image2.png)