4.4 Sum moments about the pivot.

\[ 0 = fL_1 - k_1(L_2\theta)L_2 - k_2(L_3\theta)L_3 \]

Thus

\[ f = \frac{k_1L_2^2 + k_2L_3^2}{L_1} \theta \]

But \( x = L_1\theta \), so

\[ f = \frac{k_1L_2^2 + k_2L_3^2}{L_1^2}x = k_e x \]

4.6

\[ k_e = 4 \left( \frac{EA}{L} \right) = \frac{4E\pi(d/2)^2}{L} = \frac{4(2 \times 10^{11})\pi(0.03)^2}{4(1)} = 1.8\pi \times 10^8 \text{ N/m} \]
First reduce the system to the equivalent one shown in part (a) of the figure, where

\[ \frac{1}{k_1} = \frac{1}{2k} + \frac{1}{k} = \frac{3}{2k} \]

Thus \( k_1 = 2k/3 \).

From part (b) of the figure,

\[ \frac{1}{k_e} = \frac{1}{k} + \frac{1}{k_1 + k} = \frac{k_1 + 2k}{k(k_1 + k)} \]

Thus, solving for \( k_e \) and substituting for \( k_1 \), we obtain

\[ k_e = \frac{k(k_1 + k)}{k_1 + 2k} = \frac{5k}{8} \]
4.12 Sum moments about the pivot to obtain

\[ mL_3^2 \ddot{\theta} = -(k_1 L_1 \theta) L_1 + k_2 (x - L_2 \theta) L_2 - mg L_3 \theta \]

Collecting terms we obtain

\[ mL_3^2 \ddot{\theta} + (k_1 L_1^2 + k_2 L_2^2 + mg L_3) \theta = k_2 L_2 x \]
4.17 a) Refer to the figure. Summing forces in the $x$ direction gives

$$m\ddot{x} = k(y - x) - f_t \quad (1)$$

Summing moments about the mass center of the wheel gives $I\ddot{\theta} = Rf_t$. But $x = R\theta$, and thus $\ddot{\theta} = \ddot{x}/R$. Therefore

$$f_t = \frac{I}{R}\ddot{\theta} = \frac{I}{R^2}\ddot{x} \quad (2)$$

Combine (1) and (2) to get:

$$\left(m + \frac{I}{R^2}\right)\ddot{x} + kx = ky$$

where $I = mR^2/2$. Thus

$$1.5m\ddot{x} + kx = ky \quad (3)$$

Figure 2: for Problem 4.17
4.17 continued:

b) Substituting the values into (3) gives

\[ 15\ddot{x} + 1000x = 1000y \]

or

\[ 3\ddot{x} + 200x = 200y \]

The roots are \( s = \pm 10\sqrt{2/3} \). The response to a unit-step input is

\[ x(t) = 1 - \cos 10\sqrt{\frac{2}{3}}t \]

4.21 The torsional stiffness of the torsion bar is

\[ k_T = \frac{\pi GD^4}{32L} = \frac{1.7 \times 10^9\pi(1.5/12)^4}{32(4)} = 10^4 \text{ lb - ft/rad} \]

The characteristic equation is

\[ Is^2 + k_T = 0 \]

where \( I = mL^2 = (40/32.2)(2)^2 = 4.9689 \). Thus the natural frequency is

\[ \omega_n = \sqrt{\frac{k_T}{I}} = 44.86 \text{ rad/sec} \]

4.29 The equivalent mass is \( m_e = 30/g + 0.23(7/g) = 31.61/g = 0.9817 \text{ slug} \). Note that the motor causes an additional static deflection of 0.8 in. From statics,

\[ k(0.8) = 30 \]

or \( k = 37.5 \text{ lb/in.} \)