ACCEPTANCE SAMPLING

PROBLEM STATEMENT

Acceptance sampling is often used to determine the disposition of incoming raw material or parts when 100 percent inspection is destructive, time consuming, or expensive. An effective acceptance sampling plan will allow a company to set and maintain an acceptable quality assurance standard while procuring parts or raw material from outside vendors. The scenario described below illustrates the use of acceptance sampling: A company receives a lot of raw material from its vendor. To determine whether this lot fits with the specification, a sample is taken from the lot and is inspected according to certain standards. The result of the inspection is then compared to a pre-set acceptance criterion and a decision is made accordingly. Usually, the decision is either to accept or reject the lot. An accepted lot will be used in production and a rejected lot will be returned to the vendor. The procedure involved in the above scenario applies statistical principles to specify the requirements of (1) how many units need to be inspected; and (2) how an acceptance or rejection decision shall be made.

It should be noted that acceptance sampling is not intended to improve the quality of a process and thus is not an appropriate tool to use in process control. There are different types of acceptance sampling plans such as: attribute sampling and variable sampling. This chapter presents several commonly-used acceptance sampling plans and focuses primarily on lot-by-lot acceptance sampling. In this chapter, the theories and application of these sampling methods will be described with the use of examples, diagrams, tables and graphical illustrations. It begins with a description of basic concepts in acceptance sampling and is then followed by a detailed description of various attribute sampling and variable sampling procedures. Standard sampling plans such as ANSI/ASQC Z1.4, ANSI/ASQC Z1.9 are not discussed here since the operating procedures are clearly given in their documentation published by the American Society for Quality.

Basic Concepts

Sampling can be described as the activity of taking a snap-shot of a population in order to obtain an understanding of the observed population. In acceptance sampling, inspection is performed on a sample taken from a lot of incoming materials. A decision is made concerning the disposition of the lot based on the information obtained from this sample. In general, three approached could be considered when receiving a lot: (1) no inspection, (2) 100% inspection, and (3) sampling inspection. The first approach can be taken when the quality of the vendor's process is stable and has met certain quality assurance requirements. An 100% inspection procedure may be needed when the received part or material is so critical that warrants a full inspection. In most other cases, sampling inspection could be considered. These cases include but not limited to the followings:

1. When testing is destructive; if performed, all or part of the product will be lost.
2. When the cost of 100% inspection is very high in comparison to the cost of passing a nonconforming item.
3. When the lot size is large and 100% inspection is not feasible.
4. When lots of the same material or part are received from multiple suppliers.
5. When information concerning the supplier’s quality is not available.
6. When automated inspection is not used.

Acceptance sampling itself could not help to enhance the quality of the received part or material since the quality was determined before lot was received. In a way, a rejected lot returned to the supplier might help it to improve its future quality.

Types of Sampling Plans

Sampling plans can be classified as attributes or variables plans. This classification deals mostly with the quality characteristic of interest. If the quality characteristic is not measurable but countable, attribute sampling could be considered. On the other hand, variable sampling plans might be considered if the quality characteristic is measurable. Usually, attribute sampling plan requires a larger sample size than variable sampling plans but is relatively easy to use. In attributes sampling, a predetermined number of units from each lot are inspected by which each unit is graded as conforming or nonconforming. If the number of nonconforming (i.e. a unit which does not meet product specifications for one or more quality characteristics) units is less than or equal to the prescribed minimum, the lot is accepted; other than that, it is rejected. In variable sampling, actual measurements of sample units are used in decision making rather than classifying units into conforming or nonconforming. Variable sampling plans often deal with certain statistical parameters of the submitted lot, such as the mean or the standard deviation. If the sampling statistic is found to conform to the specified acceptance limit(s), the lot is accepted; otherwise, it is rejected.

Depending on the number of samples to be taken from the lot, sampling plans can be further classified as single sampling, double sampling, multiple sampling and sequential sampling. In the attribute case, a single sampling plan requires a single sample to be taken from the lot. The lot will be accepted if the number of nonconforming units found in the sample is less than or equal to a pre-specified acceptance level, otherwise, it will be rejected. Double sampling will require a second sample to be drawn if the number of nonconforming found in the first sample fell into a certain range, otherwise, an acceptance or rejection decision will be made based on the first sample. Extending from the concept of double sampling, multiple sampling may require a series of samples to be drawn before reaching a decision. When the sample size of each sample is reduced to one, sequential sampling takes place. In sequential sampling, one item is inspected at a time and a decision could be accept, reject, or inspect another item. The process is repeated until a decision is made.

Risks in Acceptance Sampling

Due to the nature of sampling, only a portion of the lot is inspected, it always incurs certain risks. It is possible that a lot was rejected or accepted while it should not be. Often, we call these risks as the *producer's risk* and the *consumer's risk* as further formulated below.

Producer's risk = Pr{Reject a good lot },
Consumer's risk = Pr{Accept a bad lot}.

In practice, the producer's risk is denoted by α which represents the probability that a lot with an acceptable quality or better is rejected. The consumer's risk, denoted by β, represents the probability that a lot is accepted with an unacceptable quality level. The two threshold quality levels leading to opposite lot sentencing are usually referred as *acceptable quality level* (AQL) and *rejectable quality level* (RQL). When designing an acceptance sampling plan, a common goal is to allow a lot be accepted with a probability greater than (1-α) when the lot has a quality level greater than the AQL. In other words, we would like to limit the probability of rejecting an acceptable lot to α. By the same reasoning, we would desire to have an acceptance sampling plan that might mistakenly accept an unacceptable lot with a probability less than β.

Operating Characteristic Curve (OC Curve)

The operating characteristic (OC) curve is an evaluation tool that shows the probability of accepting a lot submitted with a various range of quality levels. It displays the discriminatory power of a sampling plan against various suppliers' quality levels. Several key issues of the OC curve concept is illustrated below using single sampling plans. When the lot size is finite, the probability of accepting a lot can be computed by using the hypergeometric distribution.

$$p(x \le c) = \sum_{x=0}^{c} \frac{\binom{D}{x}\binom{N-D}{n-x}}{\binom{N}{n}}$$

where, N = lot size
n = sample size
D = number of nonconforming units in the lot
x = number of nonconforming units in the sample
c = acceptance level

For example, let's consider a lot with 100 units that is subjected to inspection. An arbitrary acceptance sampling plan might be: Take a sample of 25 units from the lot, if the number of nonconforming units found is zero (c = 0), the lot will be accepted; otherwise, the lot will be rejected. If there are no nonconforming units in the lot, i.e., D = 0, then the probability of accepting the lot can be calculated as:

$$p(x \le 0) = \sum_{x=0}^{0} \frac{\binom{25}{0}\binom{100-0}{25-x}}{\binom{100}{25}} = 1$$

Since there is no nonconforming unit in the lot, the lot will always be accepted. In other words every time a lot of this quality is inspected, there is a 100% probability of acceptance. If the number of nonconforming units in the lot increases, say to 1, then the probability of accepting the lot drops to 0.75 or 75%. The relationship between the lot fraction nonconforming and the probability acceptance is further illustrated as the blue OC curve in Figure 1.

When the lot size is large, the probability of accepting a lot can be calculated by the Binomial distribution.

$$p(x \le c) = \sum_{x=0}^{c} \binom{n}{x} p^{x} (1-p)^{n-x}$$

where, n = sample size
p = lot fraction nonconforming
x = number of nonconforming units in the sample
c = acceptance level

Following the same example but assuming a large lot size, the OC curve is illustrated as the red curve in Figure 1.

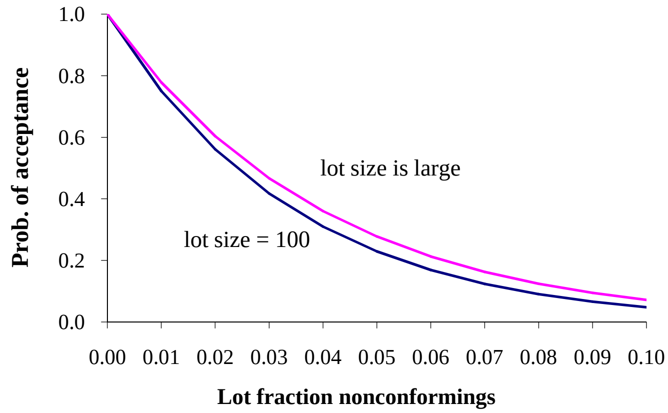

Figure 1. Examples of OC curve

Sample Size and Acceptance Level

The design of an acceptance sampling plan is largely determined by the OC curve. Ideally, it is desired to have an acceptance sampling plan that will accept a "good" lot with 100% probability while having 0% chance to accept a "bad" lot. This can not be achieved unless with 100% error-free inspection. Generally, the precision of a sampling plan increases with the size of the sample. Figure 2 below illustrates the discriminating power among the three sampling plans with different sample sizes using an acceptance level of 0 and assuming an infinite lot size. If the AQL and the RQL were set at 1% and 9% respectively, the sampling plan with a sample size of 10 will incur a producer's risk of 10% and a consumer's risk of 39% while the sampling plan with a sample size of 40 will have α=33% and β=1.4%. As seen from these curves, sampling plans with larger sample size tend to be "stricter" than those with smaller sample sizes.

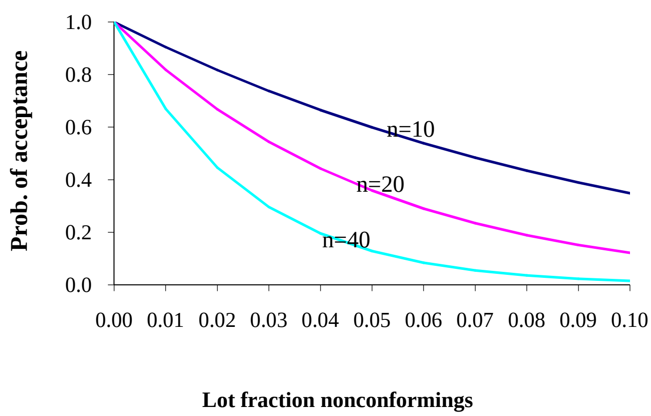

Figure 2. OC curves of various sample sizes

In addition to the sample size, the probability of acceptance could also be greatly affected by the acceptance levels. Generally speaking, the greater the acceptance level, the higher the probability of acceptance, as a consequence, the larger the consumer's risk. This is further demonstrated in Figure 3.

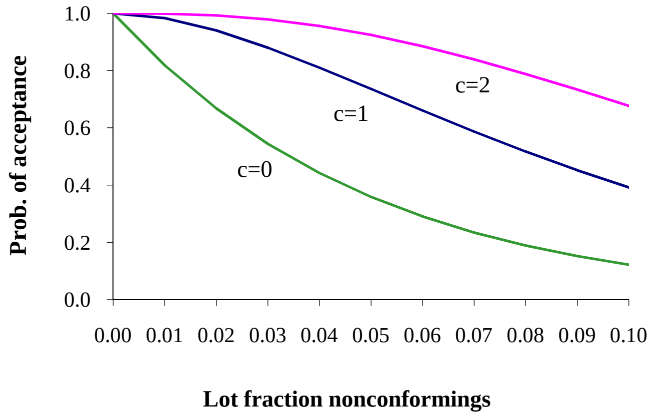

Figure 3. OC curves with the same sample size, n=20, but different acceptance levels

The choice on the sample size (*n*) and acceptance level (*c*) should be that match the goal of the user. An appropriate acceptance sampling plan can be produced with given specifications on *producer's risk* (α), *consumer's risk*(β), *acceptable quality level (AQL)*, and *rejectable quality level (RQL)*. The OC curve with the combinations of *n* and *c* that fit into the specification shall deliver the desired sampling plan. Figure 4 below illustrates these specifications w.r.t. an OC curve.

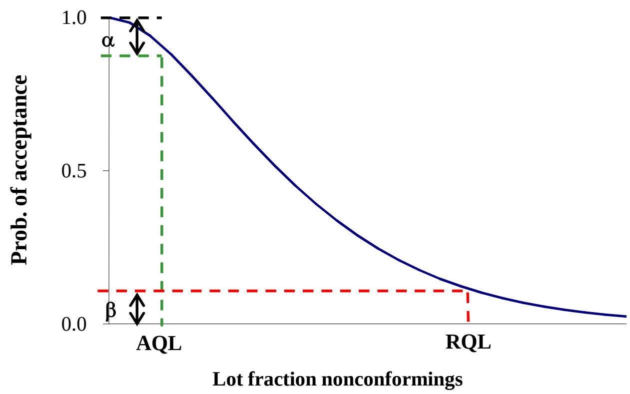

Figure 4. Specific points in an OC curve

PROBLEM SOLUTION

Acceptance sampling plans are designed to determine the disposition of submitted lots from suppliers. As illustrated in the basic concepts, the major challenge here is to find suitable sample size and acceptance level that meets certain user-specified risks and quality levels. Now we will discuss several approaches for finding the suitable sampling plans.

Acceptance Sampling by Attributes

Acceptance sampling by attributes is often employed in the cases where the quality characteristic of interest is not available in measurements. Commonly, parts or products in this type of sampling plans are classified as conforming or nonconforming units with regard to a set of given requirements or specifications. Three types of attribute sampling plans are discussed below, they are : I) single sampling plans, ii) double sampling plans, and iii) sequential sampling plans.

Single Sampling Plans for Attributes

A single sampling plan is a procedure by which a single sample is drawn form a lot and inspected. The lot is accepted if the number of nonconforming units found in the sample is less than or equal to the acceptance number (or a specified limit); otherwise, the lot is rejected. Now let's discuss an approach to find the appropriate single sampling plan.

Scenario

"A company wants to have a single sampling plan that will not accept, more than 10% of the time, material that is 8% defective or worse. In the mean time, this company would like to have at least 95% of chance to accept a submitted lot with 1% or less nonconforming."

Interpretation

AQL = p_1 = 1%, Producer's risk, α = 100% - 95% = 5%

RQL = p_2 = 8%, Consumer's risk, β = 10%

Task

Find sample size, n, and acceptance level, c.

Steps

1. Determine the ratio of p_2/p_1. Here we have the ration calculated as 8.
2. Obtain c, the acceptance level, from the table below by using the nearest ratio of p_2/p_1.

***c* acceptance level**	***p_1n*** (Pa=0.95)	***p_2n*** (Pa=0.10)	***p_2/p_1***
0	0.051	2.30	45.10
1	0.355	3.89	10.96
2	0.818	5.32	6.50
3	1.366	6.68	4.89
4	1.970	7.99	4.06
5	2.613	9.28	3.55
6	3.285	10.53	3.21
7	3.981	11.77	2.96
8	4.695	12.99	2.77
9	5.425	14.21	2.62
10	6.169	15.41	2.50
11	6.924	16.60	2.40
12	7.690	17.78	2.31
13	8.464	18.96	2.24
14	9.246	20.13	2.18
15	10.04	21.29	2.12

Table 1. Table for a two-point design of a single sampling plan with α at 0.05 and β at 0.10*

* Source: From F. E. Grubbs, "On designing single sampling inspection plans," Annals of Mathematical Statistics 20, 1949, p.256

Here, with a ratio of p_1/p_2 equal to 8, we can choose to have c either at 1 or 2.

3. Determine the sample size for each c value by holding either α or β at specified level.

   If let c = 1 and hold β at 0.10, the sample size can be calculated as:

   $$n = (p_2 n) / p_2 = 3.89/0.08 \approx 49.$$

   If use the same c value but hold α at 0.05, the sample size can be calculated as:

   $$n = (p_1 n) / p_1 = 3.55/0.01 \approx 36.$$

   If let c = 2 and hold β at 0.10, the sample size can be calculated as:

   $$n = (p_2 n) / p_2 = 5.32/0.08 \approx 67.$$

   If use the same c value but hold α at 0.05, the sample size can be calculated as:

   $$n = (p_1 n) / p_1 = 0.818/0.01 \approx 82.$$

4. State the eligible sampling plans and choose the one fit most closely to the specification. According to the above, there are four eligible sampling plans with various sample sizes and acceptance levels, they are:

   plan A- n = 49, c = 1,
   plan B- n = 36, c = 1,
   plan C- n = 67, c = 2, and
   plan D- n = 83, c = 2.

   It should be noted that plan A and C were obtained by holding their consumer's risk at 10% and plan B and D were obtained by holding their producer's risk at 5%. In order to find the most appropriate sampling plan, we need to calculate the producer's risks for plan A and C, and consumer's risks for B and D. Using plan B as an example, its consumer's risk can be calculated as:

   $$\beta = \Pr(x \le 1) = \sum_{x=0}^{1} \binom{36}{x} 0.08^{x} (1-0.08)^{36-x} = 0.21.$$

Result

The four eligible sampling plans and their respective producer's risks and consumer's risks is given in the table below.

Plan	Sample size	Acceptance level	Producer's risk	Consumer's risk
A	49	1	0.09	0.10
B	36	1	0.05	0.21
C	67	2	0.03	0.10
D	82	2	0.05	0.04

Table 2. Eligible sampling plans and associated risks.

Since the company would like to have $\alpha \leq 5\%$ and $\beta \leq 10\%$ with respect to the associated quality levels, they can choose to have either plan C or plan D. The OC curve associated with plan C is given below in Figure 5.

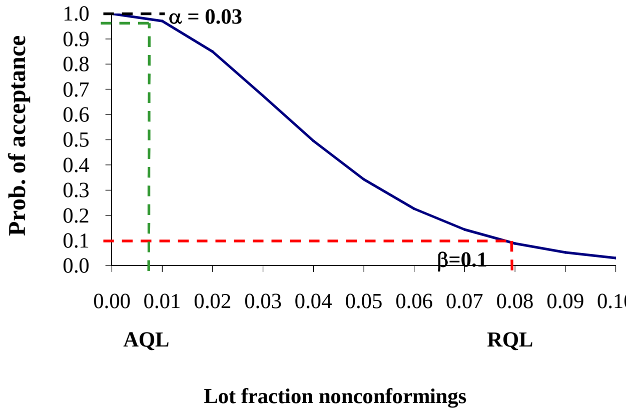

Figure 5. The OC curve of a single sampling with n = 67 and c =2

Double Sampling Plans for Attributes

A double sampling plan is an extension of a single sampling plan. In double sampling, a second sample is required to decide whether a lot should be rejected or not if the information obtained from the first sample fell into a "gray" area. As shown in Figure 6, the procedure for double sampling is to draw and inspect a random sample of size n_1 units from the lot. If the number of nonconforming units, D_1 , found in this first sample, is less than or equal to C_1 , the lot is accepted. The lot will be rejected if D_1 is greater than C_2. Otherwise, a second sample of n_2 units will be taken from the lot and inspected. If the number of nonconforming units from both samples ($D_1 + D_2$), is less than or equal to the acceptance level, C_2 , the lot is accepted; otherwise the lot is rejected.

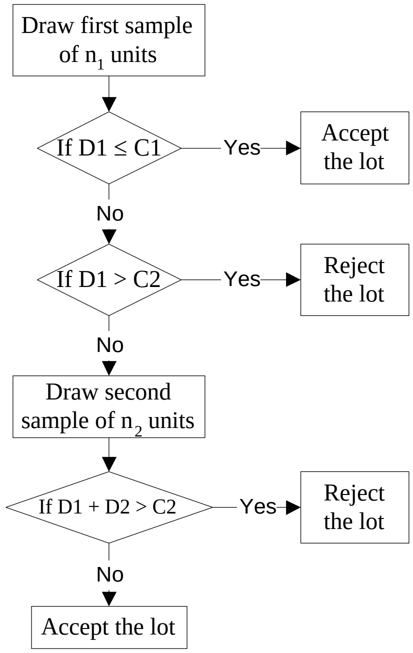

Figure 7. A flow chart for double sampling

The benefit of using a double sampling plan rather than a single sampling plan is that a double sampling plan reduces the total number of units inspected. Since the first sample size in a double sampling plan is usually smaller than that of a single sampling plan, if a decision on the basis of the first sample is made, fewer items inspected is realized. Also using a double sampling plan gives a second chance to the lot to pass inspection. However, there are also some disadvantages associated with this plan. Firstly, if a decision cannot be made on the first sample, the total number of units inspected could become larger than that of a single sampling plan. Secondly, double sampling is administratively more complex compared to that of a single sampling. Last but not least, it is possible for storage and handling problems to occur while the lot is waiting for the second sampling to be taken.

Following the same scenario as given in the single sampling plan, here we will discuss the approach of finding an appropriate double sampling plan.

Problem Requirements

AQL = p_1 = 1%, Producer's risk, α = 100% - 95% = 5%
RQL = p_2 = 8%, Consumer's risk, β = 10%
$n_2 = 2\, n_1$

Task

Find sample sizes, n_1 and n_2, and acceptance levels, c_1 and c_2.

Steps

1. Determine the ratio of p_2/p_1. Here we have the ration calculated as 8.
2. Obtain c_1, c_2, the acceptance levels, from the table below by using the nearest ratio of p_1/p_2.

***c_1 and c_2* acceptance levels**	***p_1n_1*** (Pa=0.95)	***p_2n_1*** (Pa=0.10)	***p_2/p_1***
0, 1	0.16	2.32	14.50
0, 2	0.30	2.42	8.07
1, 3	0.60	3.89	6.48
0, 3	0.49	2.64	5.39
1, 4	0.77	3.92	5.09
0, 4	0.68	2.93	4.31
1, 5	0.96	4.02	4.19
1, 6	1.16	4.17	3.60
2, 8	1.68	5.47	3.26
3, 10	2.27	6.72	2.96
3, 11	2.46	6.82	2.77
4, 13	3.07	8.05	2.62
4, 14	3.29	8.11	2.46
3, 15	3.41	7.55	2.21
4, 20	4.75	9.35	1.97
6, 30	7.45	12.96	1.74

Table 3. Table for a two-point design of a double sampling plan with α at 0.05 and β at 0.10*

* Source: Adapted from Chemical Corps Engineering Agency, Manual no. 2, Master Sampling Plans doe Single, Duplicate, Double, and Multiple Sampling, Army Chemical Center, Edgewood, Arsenal, MD, 1953.

Here, with a ratio of p_1/p_2 equal to 8, we can choose to have c_1 at 0 and c_2 at 2.

3. Determine the sample sizes by holding either α or β at specified level. If hold β at 0.10, the sample sizes can be calculated as:

$$n_1 = (p_2 n_1) / p_2 = 2.42/0.08 \approx 30$$
$$n_2 = 2\ n_1 = 60.$$

If use the same c values but hold α at 0.05, the sample sizes can be calculated as:

$$n_1 = (p_1 n_1) / p_1 = 0.30/0.01 \approx 30$$
$$n_2 = 2\ n_1 = 60.$$

Result

According to the above, there is only one eligible sampling plan which is:

$$n_1 = 30,\ n_2 = 60,\ C_1 = 0,\ C_2 = 2.$$

Computation

The probability of acceptance in a double sampling plan is the sum of the probabilities that the lot was accepted at the first and the second sample. Let P_a denotes the overall probability, P_I and P_{II} denote the individual probability, and we have

$$P_a = P_I + P_{II}$$

With the sampling plan above, P_I is the probability that $D_1 \le C_1$ observed in the first sample of 30 units. Assuming a 5% lot fraction nonconforming, P_I can be obtained as 0.215. To obtain the probability of P_{II}, we need to consider a number of possible situations. It should be noted that the second sample will not be drawn if $D_1 \le C_1$ or $D_1 > C_2$. In other words, the following conditions might exist if the lot was accepted after drawing the second sample.

1. $D_1 = 1$, $D_2 =$ 0 or 1. The probability of this can be calculated as:

$$P\{D_1 = 1, D_2 \le 1\} = P\ (D_1 = 1 \mid n_1 = 30)\ P(D_2 \le 1 \mid n_2 = 60) = (0.339)\ (0.192) = 0.065.$$

2. $D_1 = 2$, $D_2 =$ 0. The probability of this can be calculated as:

$$P\{D_1 = 2, D_2 = 0\} = P\ (D_1 = 2 \mid n_1 = 30)\ P(D_2 = 0 \mid n_2 = 60) = (0.259)\ (0.046) = 0.012.$$

The probability of acceptance at the second sample is thus the sum of the two and is obtained as 0.077. The probability of the lot acceptance, P_a, at 5% lot fraction nonconforming can then be calculated as:

$$P_a = P_I + P_{II} = 0.215 + 0.077 = 0.292.$$

OC Curve

The OC curve for a double sampling plan consists of a primary (or principal) OC curve and a supplementary curve. The primary OC curve shows the probability of acceptance on the combined first and second samples versus the lot fraction nonconforming; while that of the supplementary curve shows the probability acceptance on the first sample. Figure 8 below gives the OC curves in this illustration.

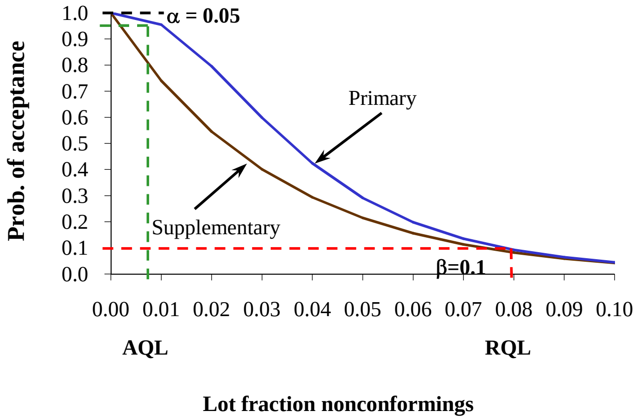

Figure 8. OC curves for a double sampling plan, $n_1 = 30$, $n_2 = 60$, $C_1 = 0$, $C_2 = 2$

Sequential Sampling Plans

When the sampling inspection is destructive or costly, sequential sampling could be considered. In sequential sampling, individual unit drawn from the lot is inspected “sequentially” until a decision could be reached. Specifically, the cumulative number of units inspected is plotted against the total nonconforming units found. If the plotted point stay in between the accept and reject zone, another unit is inspected until a decision can be made. Figure 9 illustrates this sampling process. Theoretically, this process could go indefinitely but in practice, it should be stopped if the number of units drawn from the lot reached or exceeded the sample size requirement in an equivalent single sampling plan. As illustrated in the previous sections, we will adopt the same scenario here as an example and develop an appropriate sequential sampling plan.

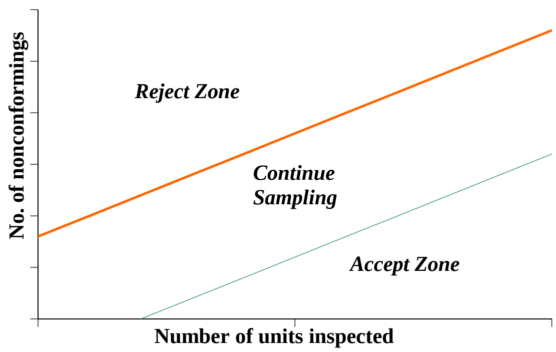

Figure 9. Decision zones in a sequential sampling plan

Problem Requirements

AQL = p_1 = 1%, Producer's risk, α = 100% - 95% = 5%

RQL = p_2 = 8%, Consumer's risk, β = 10%

Task

Determine the boarder lines between reject zone, continue zone, and accept zone.

Steps

1. Set up the graph and use the total number inspected as the horizontal axis, *n*, and the total number of nonconforming units found as the vertical axis, *D*.
2. Determine the linear functions of the two border lines.

Acceptance line, $D_{AC} = sn - h_1$,
Reject line , $D_{RE} = sn + h_2$,

where

$$h_1 = \log\frac{(1-\alpha)}{\beta} / \log\frac{p_2(1-p_1)}{p_1(1-p_2)}$$

$$h_2 = \log\frac{(1-\beta)}{\alpha} / \log\frac{p_2(1-p_1)}{p_1(1-p_2)}$$

$$s = \log\frac{(1-p_1)}{(1-p_2)} / \log\frac{p_2(1-p_1)}{p_1(1-p_2)}$$

Here we found $D_{AC} = 0.034\ n - 1.046$,
$D_{RE} = 0.034\ n + 1.343$.

Result

Once the acceptance and rejection lines were determined and plotted on a graph paper. Sample points could be dotted sequentially as individual sample unit was inspected. A decision of accept, reject, or continue sampling could be made according to the sample dot position. This process is illustrated in the following figure.

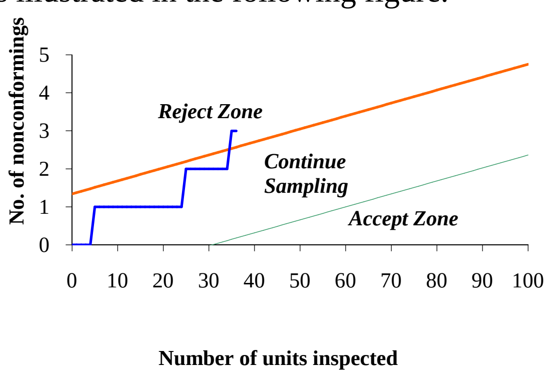

Figure 10. Graphical performance of a sequential sampling plan

Note that the inspection of sample unit 35 triggers the reject decision in this case. As can be seen in the above plan, accepting the lot is not possible until the 31st sample. In contrast, the earliest reject decision could not happen until the second sample.

OC Curve

The OC curve of a sequential sampling plan will have to pass three points, they are: (p_1, 1-α), (s, $h_2/(h_1+h_2)$), and (p_2, β). Other intermediate points could be calculated using Wald's equation:

$$p = \frac{1-(\frac{1-p_2}{1-p_1})^t}{(\frac{p_2}{p_1})^t - (\frac{1-p_2}{1-p_1})^t}$$

$$p_a = \frac{(\frac{1-\beta}{\alpha})^t - 1}{(\frac{1-\beta}{\alpha})^t - (\frac{\beta}{1-\alpha})^t}$$

with t values ranging between –2 and 2 but excluding 0. It should be noted that for t = 1, we get $p = p_1$ and $P_a = 1 - \alpha$, and for t = -1, we get $p = p_2$ and $P_a = \beta$. An OC curve for this example is exhibited below.

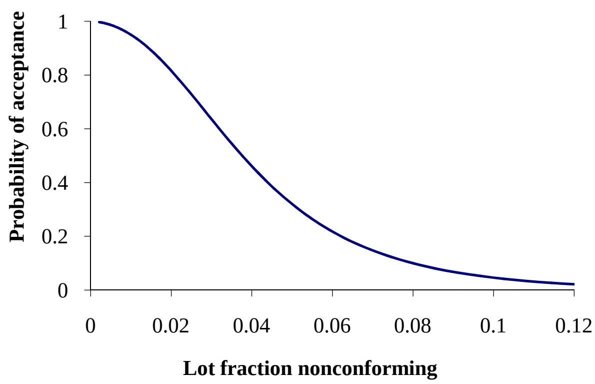

Figure 11. OC curve of a sequential sampling plan

Acceptance Sampling by Variables

Acceptance sampling by variables is often used if a quality characteristic is measured in a continuous scale following a certain type of statistical distribution. This type of plan is usually based on sample measurements on the mean and standard deviation of the lot. When using acceptance sampling by variable procedures, it often assumes that the sample measurements follow a normal distribution. Thus, a functional relationship exists between the measured quality characteristic and its mean and standard deviation. Two popular types of variable sampling plans are discussed here. They are: i) variable sampling plans for lot average quality, and ii) variable sampling plans for lot percent nonconforming.

Variables Sampling Plans for Lot Average Quality

These are mostly used in sampling products submitted in bulks (e.g. bags, boxes, and drums). This type of sampling concerns with the average quality of the part or material in the lot. It is employed to give assurance regarding the average quality of the material, instead of the fraction nonconforming. They can also be applied to assure other process parameters, such as the standard deviation of the lot. The standard technique of statistical hypothesis testing on population mean is be used to obtain sampling procedures that have specified OC curves. Again, a hypothetical scenario is given first and is followed by the development of a variable sampling plan.

Scenario

"A company wants to have a variable sampling plan that will not accept a shipment of polyester material, more than 10% of the time, if the lot average tensile strength is 95 psi or less. In the mean time, this company would like to have at least 95% of chance to accept a submitted lot with mean strength of 115 psi or more. The standard deviation of this polyester material is given as 20 psi."

Interpretation

AQL = 115 psi, RQL = 95 psi, α = 5%, β = 10%

Task

Find sample size, n, and acceptance level, X_A.

Steps

1. Formulate the problem in terms of standard normal probabilistic statements.

$$Z_\alpha = Z_{0.05} = \frac{X_A - AQL}{\sigma/\sqrt{n}} = \frac{X_A - 115}{20/\sqrt{n}} = -1.645$$

$$Z_\beta = Z_{0.10} = \frac{X_A - RQL}{\sigma/\sqrt{n}} = \frac{X_A - 95}{20/\sqrt{n}} = 1.282$$

2. Solve the two equations simultaneously for X_A and n.

$$X_A = \frac{Z_\beta \cdot AQL - Z_\alpha \cdot RQL}{Z_\beta - Z_\alpha} = \frac{1.282 \cdot 115 - (-1.645) \cdot 95}{1.282 - (-1.645)} = 103.76$$

$$n = [\frac{(Z_\beta - Z_\alpha) \cdot \sigma}{AQL - RQL}]^2 = [\frac{(1.282 - (-1.645)) \cdot 20}{115 - 95}]^2 = 8.57 \approx 9$$

Result

A variable sampling plan is obtained with an acceptance level X_A of 103.76 and a sample size of 9. This sampling plan works as follows. A random sample of 9 inspection units should be selected from the batch and have its average tensile strength calculated. If the sample has an average tensile strength less than 103.76, the lot is rejected; otherwise, it is accepted. Figure 12 gives the relationship of X_A with respect to AQL and RQL in this sampling plan.

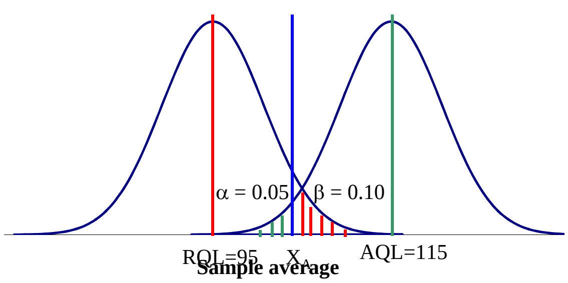

Figure 12. Relationships among AQL, RQL, and X_A

It should be noted that the procedure exhibited above is based on a known population standard deviation. In case that the population standard deviation is not given, it has to be estimated by a pre-selected sample.

Variable Sampling Plans for Lot Fraction Nonconforming

Sometimes, sampling plans might be required to measure the lot or process fraction nonconforming while the quality characteristic is a variable. In these cases, there is either a lower specification limit (LSL), an upper specification limit (USL), or both, that define the acceptable values of the parameter. If a lower specification limit is given, a standard normal deviate can be obtained as:

$$Z_{LSL} = \frac{\bar{x} - LSL}{\sigma}$$

The standard normal deviate w.r.t. the upper specification limit can be found as:

$$Z_{USL} = \frac{USL - \bar{x}}{\sigma}$$

An estimate of the lot fraction nonconforming is the area under the standard normal curve that falls outside of Z_{LSL}, Z_{USL}, or both. It should be noted that as the lot average moves away from the specification limit, the lot fraction nonconforming decreases. Figure 13 illustrates the above by using a single lower specification limit.

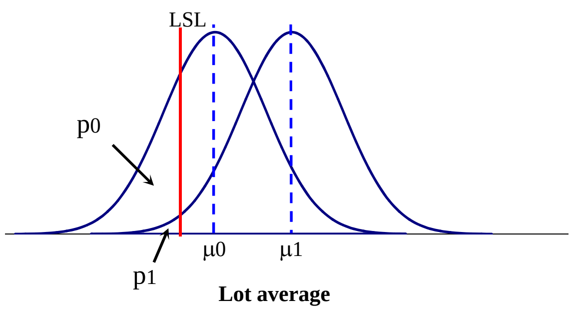

Figure 13. Relationships between lot average and lot fraction nonconforming

To determine the disposition of a lot, the standard normal deviate Z_{LSL} or Z_{USL} is compared to a critical value *k*. If it is greater than or equal to *k*, the lot is accepted; otherwise, it is rejected. The development and operation of this type of variables sampling plan is illustrated below using a similar scenario adapted from the previous section.

Scenario

"A company wants to have a variable sampling plan that can be used to determine the disposition of lots of polyester material which has a lower specification limit of 90 psi. The plan shall not accept, more than 10% of the time, a lot with a fraction nonconforming that is 8% or more. In the mean time, it would like to have at least 95% of chance to accept a submitted lot with a fraction nonconforming of 1% or less. The standard deviation of this polyester material is given as 20 psi."

Interpretation

LSL = 90 psi, AQL = p_1 = 1%, RQL = p_2 = 8%, α = 5%, β = 10%

Task

Find the sample size *n* and the critical value *k*.

Steps

1. Formulate the problem in terms of standard normal probabilistic statements.
2.

$$Z_{LSL} = \frac{\bar{x} - LSL}{\sigma}$$

Regarding to AQL, p_1, the lot will be accepted with 1-α probability if $Z_{LSL} > k$. Let μ_1 be the lot average while it has a fraction nonconforming of p_1. By the same reason, if the lot fraction nonconforming is at its RQL, p_2, it can be accepted only with a probability of β if $Z_{LSL} > k$. Let μ_2 demotes the lot average while its fraction nonconforming is at p_2. Figure 14 explains the above relationships.

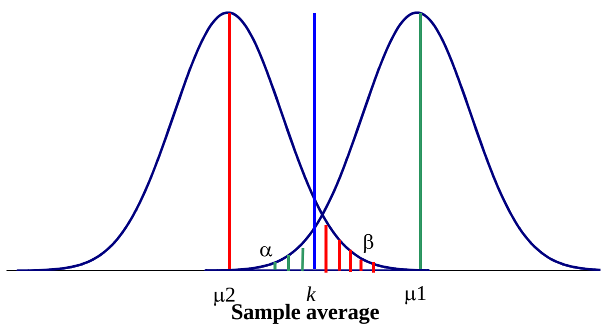

Figure 14. Relationships among sample averages and the critical value k

Let $Z_1 = (\mu_1 - LSL)/\sigma$ and $Z_2 = (\mu_2 - LSL)/\sigma$, we can obtain their corresponding values from the standard normal table where $Z_1 = 2.325$ and $Z_2 = 1.405$. Using these equations and their relationships with α and β, we can obtain:

$$Z_{1-\alpha} = -Z_\alpha = (k - Z_1)\sqrt{n}$$
$$Z_\beta = (k - Z_2)\sqrt{n}$$

3. Solve the two equations simultaneously for *n* and *k*.

$$n = [\frac{Z_\alpha + Z_\beta}{Z_1 - Z_2}]^2 = [\frac{1.645 + 1.282}{2.325 - 1.405}]^2 = 10.12 \approx 10$$
$$k = Z_1 - \frac{Z_\alpha}{\sqrt{n}} = 2.325 - \frac{1.645}{\sqrt{10}} = 1.805$$

Result

A variable sampling plan for lot fraction nonconforming is obtained with a critical value of 1.805 and a sample size of 10. This sampling plan works as follows. A random sample of 10 units should be selected from the batch and have its sample average tensile strength calculated. The standard normal deviate, Z_{LSL}, is next calculated and is compared to k. Accept the lot if Z_{LSL} is greater than *k*; otherwise, reject it.

Summary

This chapter presents the theories and applications of a variety of acceptance sampling plans for attribute and variable inspection. Compared to 100% inspection, acceptance sampling has several advantages, they include:

- Less inspection effort,
- Less disturbance on the submitted lot,
- Less cost and fewer inspectors.
- Less inventory space and storage time.

On the other hand, acceptance sampling also has some disadvantages.

- Risks involved in accepting a bad lot or rejecting a good lot ,
- Compounded effects on sampling inspection errors,
- Difficulties in select an appropriate plan.

Comparing the variable sampling plans with the attribute sampling plans, the former provides an efficient decision-making mechanism with smaller sample sizes. It is usually more accurate than attribute sampling. However, its complexity involved in the actual application often lead the user back to the simpler attribute samplings. To choose a proper acceptance sampling method, users need to carefully consider the requirements and risks involved in sampling including: the cost of measurement, the required accuracy of decision, the facility available, the imposed time constraint, and the end users' statistics background. The underlying rule here is that acceptance sampling should only be applied in situations where other effective measures of the supplier’s quality are not available. Acceptance sampling activities should be gradually phased out from a company as a more mature and stable relationship is developed with the supplier.