3D depth-limited breaking waves in fully non-linear potential flow

Sunil Mohanlal[1], **Jeffrey C. Harris**[1], **Marissa L. Yates**[1], **Stephan T. Grilli**[2]
(1) LHSV, Ecole des Ponts, EDF R&D, Chatou, France
(2) Department of Ocean Engineering, University of Rhode Island, Narragansett, RI, USA
sunil.mohanlal@enpc.fr

HIGHLIGHTS
A new method of modeling 3D depth-limited breaking waves is proposed and implemented in a fully non-linear potential flow model based on the boundary element method. The method is implemented in three steps: (1) identification of breaking onset using the kinematic B criterion; (2) application of damping pressure in the dynamic free surface condition, function of crest kinematics; and (3) a breaking termination criterion to stop this dissipation. A validation case for a numerical experiment set-up is presented.

1 INTRODUCTION
The study of breaking waves is crucial to establish engineering wave properties in complex sea states, which govern, among other things, wave interaction with fixed and floating structures. Extensive research has being done on understanding the many aspects of this phenomenon (e.g., Duncan [1], Stive [2], Banner and Peregrine [3], Barthelemy et al. [4], Derakhti et al. [5], Derakhti et al. [6]). Due to the computational complexity of modeling breaking waves over large domains in Navier–Stokes models, researchers still rely on using simpler models in which the effects of breaking waves are explicitly introduced. This was done in 2D with a variety of advanced models and methods, e.g., by Guignard and Grilli [7], Kennedy et al. [8], Simon et al. [9], Papoutsellis et al. [10], and Mohanlal et al. [11]. In 3D, however, numerical techniques have mostly been simpler and limited to preventing numerical instabilities in the model when wave breaking occurs (e.g., Pierella et al. [12], Ghadirian et al. [13]). Here, we propose a new method for modeling 3D depth-limited breaking waves, which is an extension of our earlier work in 2D (Mohanlal et al. [11]).

2 FULLY NON LINEAR POTENTIAL FLOW (FNPF) MODEL
The considered FNPF model assumes the fluid flow to be inviscid and irrotational such that the flow velocity can be written as $\mathbf{V} = \nabla\phi$, where ϕ is a velocity potential, such that $\nabla^2\phi = 0$. We use the model of Harris et al. [14], in which, as in Grilli et al. [15], Laplace's equation is solved as a boundary integral equation, discretized with a higher-order BEM,

$$\alpha(\mathbf{x}_i)\phi(\mathbf{x}_i) = \int_\Gamma \left\{ \frac{\partial\phi}{\partial n}(\mathbf{x})G(\mathbf{x}-\mathbf{x}_i) - \phi(\mathbf{x})\frac{\partial G}{\partial n}(\mathbf{x}-\mathbf{x}_i) \right\} \mathrm{d}\Gamma, \tag{1}$$

where Γ is the boundary, α is the interior solid angle at the boundary at point $\mathbf{x}_i$, and $G(\mathbf{x}, \mathbf{x}_i) = 1/(4\pi r_i)$ is the 3D free space Green's function (with $r_i = |\mathbf{x} - \mathbf{x}_i|$).

3 WAVE BREAKING MODEL
To demonstrate the breaking model, a simple 3D submerged bar (Fig. 1a) is considered,

with an incident solitary wave of relative height $H/h = 0.7$ (as in Antuono et al. [16]). Wave breaking is modeled in three steps: (1) wave crests reaching breaking onset are identified with the universal criterion that an evolving crest, whose ratio of horizontal particle velocity at the crest, u, to crest velocity, c, exceeds a critical value, $B = u/c = 0.85$, will always break; and otherwise it will not (e.g., Derakhti et al. [6]); (2) an absorbing pressure is applied to breaking crest regions [7, 17]; (3) absorption is terminated when $B_{off} = 0.3$, as in Mohanlal et al. [11], who found this to be optimal for 2D wave breaking on submerged bars, based on a few test cases.

To detect breaking crests in a general way, local maxima are first found (Fig. 1a,b), then, the surrounding 16 BEM nodes are fitted with a bi-cubic fit (see Fig. 1b; Grilli et al. [15]) in which a wave crest line segment is calculated (Fig. 1c), defined by a length (δ), mean position ($\overline{x_c}$, $\overline{y_c}$), and horizontal flow velocity at the surface, $u = \sqrt{u_x^2 + u_y^2}$ at this mean position, slope, and intercept. These crest segments are tracked in time by assuming that, for a small time step, they move approximately in the local normal direction. Phase speed along each crest segment is finally calculated as $c = \sqrt{(d\overline{x_c}/dt)^2 + (d\overline{y_c}/dt))^2}$, which yields $B = u/c$. Fig. 2 shows positions and B values of detected crests, up to breaking onset.

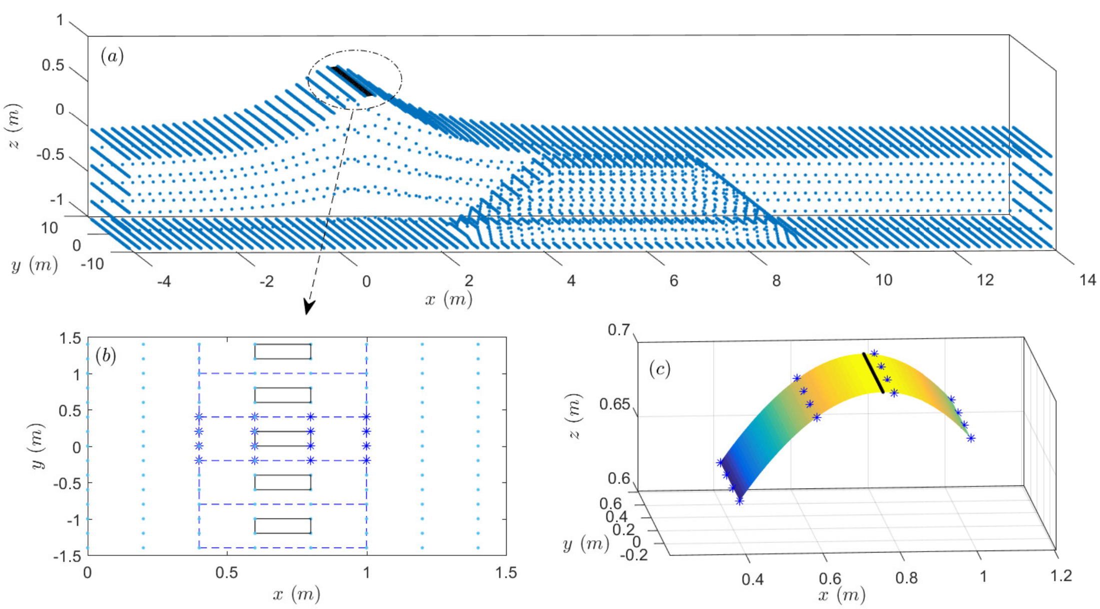

Figure 1: (a) Solitary wave propagating over 3D submerged bar ($h = 1$ m; depth near the wavemaker); dots indicate BEM nodes. (b) Close-up top view of the free surface nodes around the crest, with solid rectangles indicating the selected elements for further analysis. Nodes selected around an element for crest detection are marked as stars. (c) Bi-cubic fit on 16 nodes around selected element, with detected crest shown as black line.

The energy dissipation in breaking crests is then determined as: (1) the non-dimensional breaking strength parameter b (defined such that wave energy dissipation rate per unit length of the breaking crest, $\epsilon = b\rho g^{-1} c^5$) is determined following Mohanlal et al. [11]; and (2) $b = 0.05$ is used to calculate the instantaneous power dissipated per unit length of crest Π_b, modeled as the work over one time step of a damping pressure P_b specified in the dynamic

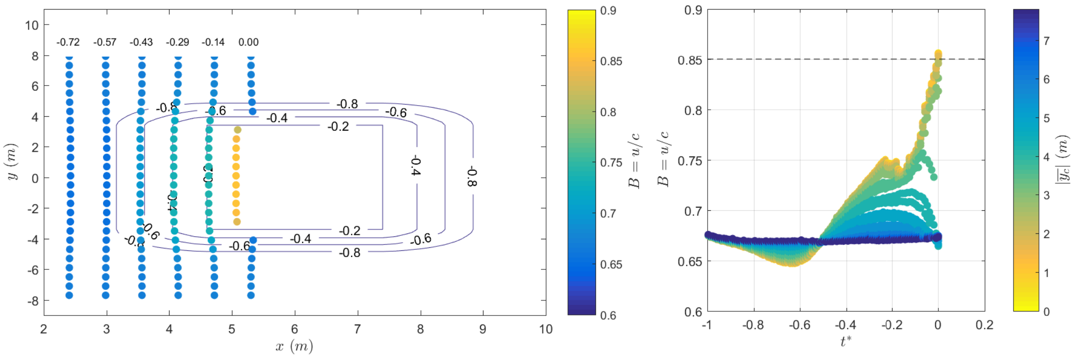

Figure 2: Left: Solitary wave crests ($\overline{x_c}$, $\overline{y_c}$) at six times $t^* = (t - t_b)/\sqrt{gh}$, with breaking onset at t_b; color scale is $B = u/c$; bottom contours shown as black lines (in meter). Right: B *vs* t^* for all crests, up to breaking onset ($B = 0.85$); color scale is $|\overline{y_c}|$ (m).

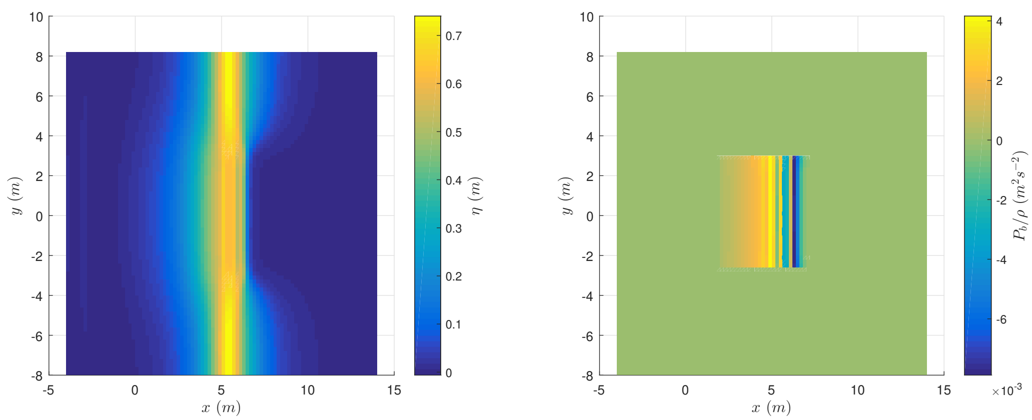

Figure 3: Left: a plot of the free surface at the breaking onset time t_b. Right: the damping pressure (P_b/ρ) applied on a section of the free surface. Note: for stability in more general cases, this should be smoothed between breaking and non breaking regions, not shown here.

free surface condition around the breaking wave crests (Grilli et al. [17]), with $P_b(x, y, t) = \nu(t)\phi_n(x, y, t)$, where the absorbing function is $\nu(t) = \Pi_b\delta/(\int_x \int_y \phi_n^2 \sqrt{1 + \eta_x^2 + \eta_y^2} dxdy)$, ϕ_n is the normal surface velocity. Fig. 3 shows computed surface and pressure at breaking onset time t_b.

4 SUMMARY

A method of modeling depth-limited breaking waves in a 3D FNPF-BEM model is demonstrated. The approach is easily extendable to other evolving wave crests, for regular waves or more realistic sea-states, as was done in earlier 2D work [11]. Numerical developments are in preparation for comparisons with existing experimental data from the literature, e.g., the free surface elevation post-breaking, and identification of wave breaking regions.

ACKNOWLEDGEMENTS

This research was produced within the framework of Energy4Climate Interdisciplinary Center (E4C) of IP Paris and Ecole des Ponts ParisTech. This research was supported by 3rd

Programme d'Investissements d'Avenir [ANR-18-EUR-0006-02]. This action benefited from the support of the Chair «Challenging Technology for Responsible Energy» led by l'X – Ecole Polytechnique and the Fondation de l'Ecole Polytechnique, sponsored by Total Energies. SG is gratefully acknowledging support from the US National Science Foundation grant #OCE-19-47960. The authors also thank Luc Pastur (ENSTA Paris) and Christophe Peyrard (EDF R&D LNHE) for helpful discussions.

REFERENCES

[1] Duncan, J. H. 1983. *The breaking and non-breaking wave resistance of a two-dimensional hydrofoil. J. Fluid Mech. 126*, 507–520.

[2] Stive, M. 1984. *Energy dissipation in waves breaking on gentle slopes.* Coastal Engng. 8(2), 99–127.

[3] Banner, M. L., and Peregrine, D. H. jan 1993. *Wave Breaking in Deep Water.* Annual Review of Fluid Mech. 25(1), 373–397.

[4] Barthelemy, X., Banner, M., Peirson, W., Fedele, F., Allis, M., and Dias, F. 2018. *On a unified breaking onset threshold for gravity waves in deep and intermediate depth water. J. Fluid Mech. 841*, 463–488.

[5] Derakhti, M., Banner, M. L., and Kirby, J. T. jun 2018. *Predicting the breaking strength of gravity water waves in deep and intermediate depth. J. Fluid Mech. 848*.

[6] Derakhti, M., Kirby, J. T., Banner, M. L., Grilli, S. T., and Thomson, J. 2020. *A Unified Breaking Onset Criterion for Surface Gravity Water Waves in Arbitrary Depth.* J. Geophys. Res.: Oceans 125(7).

[7] Guignard, S., and Grilli, S. T. Modeling of wave shoaling in a 2D-NWT using a spilling breaker model. In *The Eleventh International Offshore and Polar Engineering Conference* (2001), OnePetro.

[8] Kennedy, A. B., Chen, Q., Kirby, J. T., and Dalrymple, R. A. 2000. *Boussinesq modeling of wave transformation, breaking, and runup. I: 1D.* J. Waterway, Port, Coastal, and Ocean Engng. 126(1), 39–47.

[9] Simon, B., Papoutsellis, C. E., Benoit, M., and Yates, M. L. 2019. *Comparing methods of modeling depth-induced breaking of irregular waves with a fully nonlinear potential flow approach.* J. Ocean Engineering and Marine Energy 5(4), 365–383.

[10] Papoutsellis, C. E., Yates, M. L., Simon, B., and Benoit, M. 2019. *Modelling of depth-induced wave breaking in a fully nonlinear free-surface potential flow model. Coastal Engng. 154*, 103579.

[11] Mohanlal, S., Harris, J., Yates, M., and Grilli, S. 2022. *Unified depth-limited wave breaking detection and dissipation in fully nonlinear potential flow models. Coastal Engng. Submitted*.

[12] Pierella, F., Lindberg, O., Bredmose, H., Bingham, H. B., Read, R. W., and Engsig-Karup, A. P. 2021. *The DeRisk database: Extreme design waves for offshore wind turbines. Marine Structures 80*, 103046.

[13] Ghadirian, A., Pierella, F., and Bredmose, H. 2023. *Calculation of slamming wave loads on monopiles using fully nonlinear kinematics and a pressure impulse model. Coastal Engng. 179*, 104219.

[14] Harris, J. C., Dombre, E., Benoit, M., Grilli, S. T., and Kuznetsov, K. I. 2022. *Nonlinear time-domain wave-structure interaction: a parallel fast integral equation approach.* Intl. J. Numer. Meth. Fluids 94(2), 188–222.

[15] Grilli, S. T., Guyenne, P., and Dias, F. 2001. *A fully non-linear model for three-dimensional overturning waves over an arbitrary bottom.* Intl. J. Numer. Meth. Fluids 35(7), 829–867.

[16] Antuono, M., Lucarelli, A., Bardazzi, A., and Brocchini, M. 2022. *A wave-breaking model for the depth-semi-averaged equations. J. Fluid Mech. 948*, A50.

[17] Grilli, S. T., Horrillo, J., and Guignard, S. 2020. *Fully nonlinear potential flow simulations of wave shoaling over slopes: Spilling breaker model and integral wave properties.* Water Waves 2(2), 263–297.