# THREE-DIMENSIONAL WAVE FOCUSING IN FULLY NONLINEAR WAVE MODELS

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**Abstract:** Wave frequency focusing has been used in two-dimensional (2D) laboratory wave tanks to simulate very large waves at sea, by producing large energy concentration at one point of space and time. Here, three-dimensional (3D) frequency/directonal energy focusing is simulated in a fully nonlinear wave model (Numerical Wave Tank; NWT), and shown to produce very large waves. This method alone, however, cannot explain why and how large waves occur in nature. Self-focusing, i.e., the slow growth of 3D disturbances in an initially regular wave train, is shown to also play a major role in the formation of "freak waves". Self-focusing is studied in a more efficient space-periodic nonlinear model, in which long term wave propagation can be simulated. The combination of directional/frequency focusing and self-focusing, and resulting characteristics of large waves produced, could be studied within the same NWT.

## INTRODUCTION

The existence of abnormally large waves at sea and the understanding of physical phenomena creating them have received increasing attention in recent years. Early reports by Mallory (1974) described a long series of naval accidents caused by unexpectedly large waves. Since then, many authors have given a considerable attention to the study of large transient waves, with the aim of understanding possible physical mechanisms determining when and how these are generated. The goal is to calculate kinematics and dynamics of such wave events and, eventually, to provide models for better designing vessels and off-shore structures. So far, a number of mechanisms have been proposed for the generation of such steep wave events, but it seems that these are still poorly understood. The consensus, however, is that all of these mechanisms require to model nonlinear behavior of ocean waves.

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In some situations, the occurrence of giant waves can be explained by the focusing of wave energy due to the presence of ocean currents or the bottom topography. This is typical of some areas around the world (such as the famous 'Agulhas Current', which is responsible for the formation of freak waves off the South-East Coast of Africa). Why giant waves are generated in the open ocean, far away from non-uniform currents or bathymetry, however, is still very much an open problem. In past research efforts, the concept of 'phasing' was often used as an explanation, whereas a short-lived large wave occurred when waves in an irregular sea combined their phase at one spatial point and at a particular time. Although this concept has been used in fairly ingenious ways (Boccotti 1981), it is still usually defined within the limits of linear wave theory. Many experimental and observational results, however, have shown that, when considering rarely occurring waves, the processes are far from being stationary Gaussian ones. In other words, the Rayleigh distribution of wave heights (based on a linear representation of the sea surface) does not predict that waves as large as 2.2 to 2.4 the significative wave height—as observed at sea—will normally occur (Wolfram 2001). Such steep wave events appear to belong to a non-Gaussian distribution of rare events (Skourup et al. 1997; Haver and Andersen 2000). Therefore, when dealing with extreme waves, full nonlinearity should, in principle, be kept in the equations, since nonlinear (i.e., non-Gaussian) wave interaction processes will likely play a dominant role.

More specifically, nonlinerarity seems to act in two main ways : (i) a trivial nonlinear superposition mechanism (Dean 1990), and (ii) a more complex nonlinear instability process. The latter way will be detailed in a following section. The former way has been widely used, mainly by naval architects and off-shore engineers, who developed (mostly experimental) techniques referred to as "wave focusing", to produce extremely large waves. In practice, in a laboratory or in a numerical wave tank, wave phases are calculated to produce a large 'design' wave at a given location and, sometimes, to study the interaction of a huge wave with vessels, piles, or other mobile or fixed structures. Nonlinearity usually makes it hard to produce the highest wave at a pre-determined point, due to amplitude dispersion effects. In traditional wave focusing techniques, waves having different frequencies are focused to produce a single large wave at one prescribed time and location : the basic idea is to first generate shorter waves, followed by longer ones which, due to frequency dispersion, are faster and catch up with the shorter waves over a some small area of space, thus producing a particularly high and steep wave through superposition (Chaplin 1996). This focusing method, however, is mostly limited to unidirectional situations and used in laboratory wave tanks to simulate all kinds of conditions, from slightly spilling breakers (Schlurmann et al. 2000) to violent plunging breakers (Dommermuth et al. 1988). Three-dimensional effects, which can be very important, have often been neglected, mainly because 3D wave tanks are very costly to operate, and 3D wave generation is also a difficult task. Observations show, however, directional focusing effects associated with 3D features of the wave field, such as a continuous curvature of the wave front,

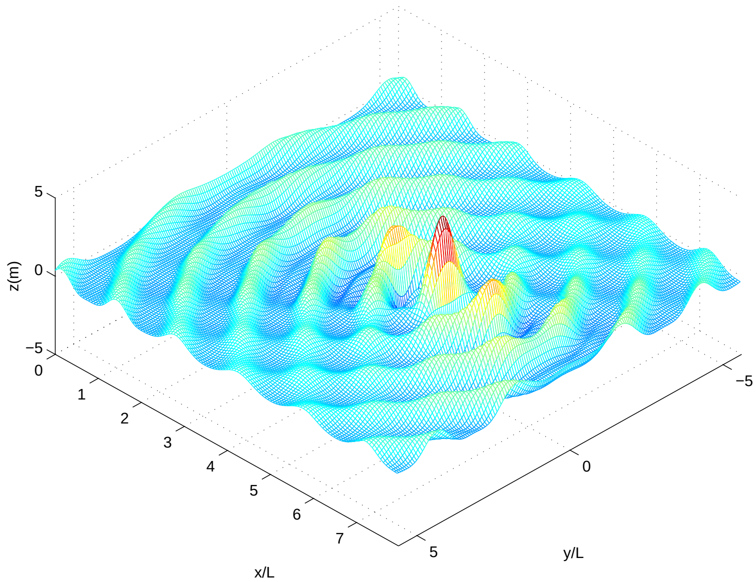


**FIG. 1. Example of second-order directional wave focusing**

cannot be neglected in the analysis of extreme waves at sea (She et al. 1997, Nepf et al. 1998). Experiments have shown that curved wave fronts lead to 3D breaking waves, and that the shape and kinematics of 3D breaking waves may greatly differ from those of 2D breakers. The degree of angular spreading is found to have great effects on wave breaking characteristics and kinematics, and, hence, non-directional wave theories are demonstrated to be insufficient to describe the kinematics of 3D waves.

## LOW-ORDER 3D WAVE FOCUSING

A ready-to-use solution for wave focusing may be easily obtained using low-order wave theories. Curved wave fronts are generated by using a number of wave fronts of the same height and frequency, equally spaced within an (horizontal) angular range. The phases $\varphi$ of the fronts are calculated so that energy becomes focussed at a predetermined point

$$\varphi = k(x - d_f)\,\cos\theta \,+\, ky\,\sin\theta \,-\, \omega t \qquad (1)$$

where $d_f$ is the focal distance (in the $x$ direction) and $-\alpha \leq \theta \leq \alpha$. To accelerate focusing, one can impose an additional frequency-focusing, by adjusting the wave frequency as a function of the angle of incidence $\theta$, thus increasing the curvature of wave fronts. For mild incident waves, this can approximately be done based on the linear dispersion relationship. [Thus, if $k$ denotes the wavenumber for $\theta = 0$ and frequency $\omega$, and $k_\theta = k\cos\theta$ is the wavenumber for angle $\theta$, then frequency may be slightly changed to satisfy the linear dispersion relationship, $\omega_\theta^2/g = k_\theta \tanh k_\theta h$.]

Up to second-order, it is possible to obtain an analytical solution for directional focusing, considering all 2nd-order interaction terms, as given, e.g., by Hu

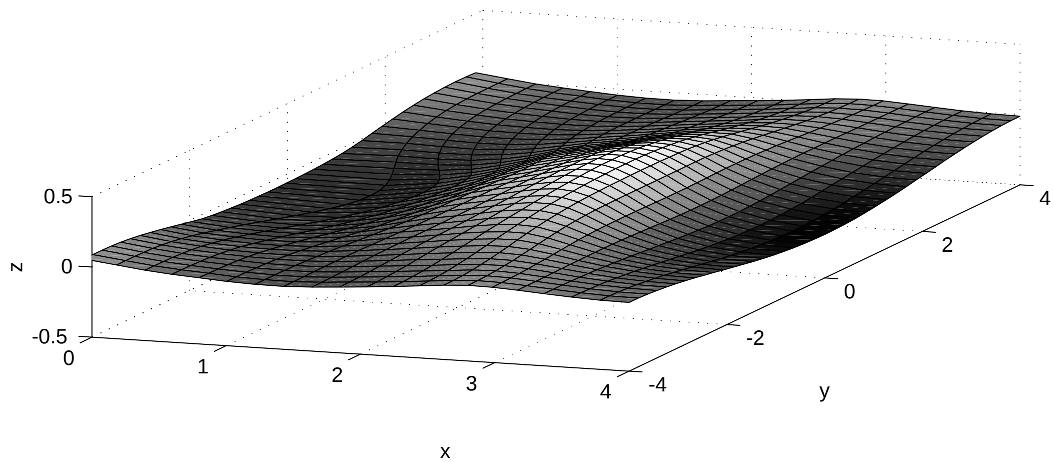


**FIG. 2. Directional focusing in a NWT : a non-breaking case**

(1996) (Fig. 1). Nonlinear wave-wave interactions become only important near the focusing point while far from this point, linear theory and second-order theory give nearly the same results (Brandini and Grilli 2000,Brandini and Grilli 2001a).

## FULLY NONLINEAR 3D WAVE FOCUSING

To generate a focussed signal in a wave tank, such as shown in Fig. 1, one needs a wave generation system that can specify wave propagation from many directions. This is achieved using directional or "snake" wavemakers, which are long articulated wavemakers consisting of numerous wave paddles, that can be moved independently from one another. The linear solution for directional wave focusing in a wavetank equipped this way, and having impermeable (reflective) lateral walls, was derived by Dalrymple (1989). A 3D fully nonlinear potential flow model (i.e., NWT), based on a Boundary Element Method (BEM) with an Eulerian-Lagrangian flow representation, was recently developed by Grilli et al. (2001a) (also see, Grilli et al. 2001b). Extension of this NWT to model 3D directional wave focusing, including the additional possibility of frequency-focusing, was done by Brandini and Grilli (2001b). A snake wavemaker, similar to those used in laboratory facilities, was modeled at one extremity of the 3D-NWT, and a new open boundary condition, based on a snake absorbing wavemaker, was modeled at the other extremity. The snake wavemaker motion was prescribed according to linear wave theory (Dalrymple 1989), such as to generate curved wave fronts and focus wave energy at a specified distance $d_f$ away from the wavemaker. The image method and symmetrical properties of the solution were implemented, to reduce the size of the computational domain in the BEM, and hence the computational cost. Figs. 2 and 3 illustrate such computations. In both cases, we see the generation of a curved wave front having a very high elevation around $y = 0$. In Fig. 3, the wave starts spilling breaking at the crest, which interrupts computations.

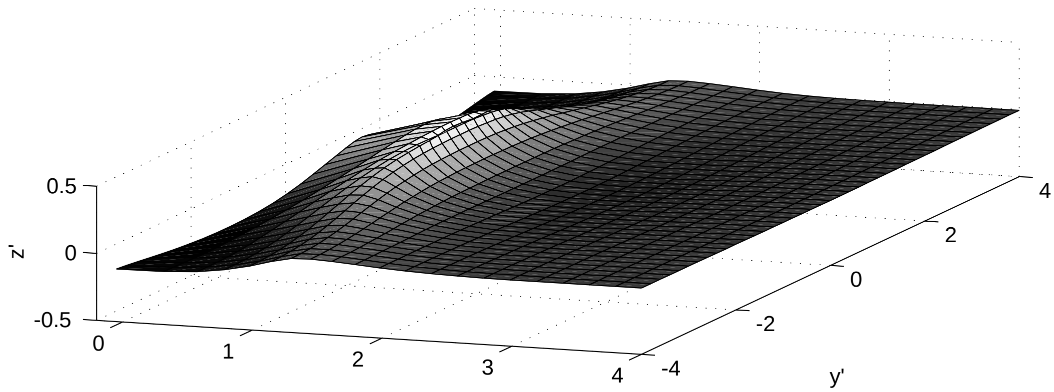


**FIG. 3. Directional focusing in a NWT : a breaking case**

## 3D SELF-FOCUSSED WAVES

### Wave instability to small modulations

Although many solutions are theoretically possible, based on a 'phasing' concept, the probability of finding a large enough number of waves, in a random wave field, whose phases would match at the same wave crest, is extremely low. Thus, research was done in recent years to discover which other physical mechanism might be responsible for the generation of large wave energy concentration at one point in the ocean. Many researchers concentrated their efforts on wave instability phenomena. The pioneering work of Benjamin and Feir (1967) on the instability of periodic waves of finite amplitude, caused a small revolution at the time. In water of sufficiently depth, Benjamin-Feir's (BF) theory predicts that a slightly modulated 2D periodic wave train will evolve into strongly modulated wave groups, where the wave of maximum amplitude may be much larger than that of the original wave train. Since even a relatively regular ocean swell contains many frequencies, according to BF's theory, a perfectly regular time-harmonic wave train can therefore never exist. The phenomenon of 'natural' evolution of periodic waves into a series of wave groups has been referred to as 'self-focusing'. Many authors suggested that BF instability is the mechanism explaining the formation of waves much larger than expected. However, other instability phenomena have been identified. McLean (1982) theoretically predicted a type of wave instability (called type II), which is predominantly 3D, while BF (called type I) is only 2D. Su et al. (1982) experimentally confirmed this prediction by showing how a steep 2D wave train can evolve into 3D spilling breakers.

Type I and II instabilities involve nonlinear effects. In fact, they can only be identified by developing evolution equations at least to the third-order (such as the nonlinear Shrödinger equation or its modifications, e.g., Henderson et al. 1999, Trulsen and Dysthe 1999). Henderson et al. (1999) also performed fully nonlinear calculations to study the behaviour of 2D uniform wave trains of moderate steepness, perturbed by a small periodic perturbation. After a large time of propagation (typically over 100 wave periods), it is observed that a large steep wave (i.e., a "freak wave"), may emerge from the initial wave train, and break or recede, and periodically reappear.

**Self-focusing in a 3D model**

In previous numerical studies, 3D effects were not usually addressed because, either it was not possible to generalize the method of solution to 3D, or the computational effort in a 3D model was too high. In the present work, we adopt the computationally efficient Higher Order Spectral (HOS) method, independently developed by West et al. (1987) and Dommermuth and Yue (1987). This rapidly convergent method represents the sea surface as a modal (Fourier) superposition, by way of a perturbation expansion. Doubly periodic boundary conditions are specified in the horizontal plane. HOS allows computations at any desired order in nonlinearity. Here, we use a fourth-order method. We start with an initial quasi-2D wave train (modeled as a streamfunction wave), with small initial periodic perturbations in both the longitudinal $x$ direction (as it has been done to show the BF instability) and the lateral one $y$. Cases are characterized by the initial steepness of the wave train $ak$, and two characteristic modulation wavelengths ($l_x$ and $l_y$). Computations are carried out for many wave periods, because a strong growth of instabilities only appears after about 100 wave periods.

The evolution of a modulated wavetrain with $ak = 0.14$, $l_x = 5$ and $l_y = 10$ (hence with a lateral modulational wavelength twice the longitudinal one) is shown in Fig. 4. While at earlier stages of evolution waves are essentially 2D, at later stages, the growth of transverse perturbations causes a 3D structure to develop. At final stages, both a longitudinal and a transverse growth of such modulations is observed. Fig. 4a shows the evolution at time $t/T = 90$ (with $T$ the wave period). We see the combination of two effects :

- In the longitudinal direction, a BF-like mechanism causes the wave group to shorten ahead and to lengthen behind, with a wave energy concentration in the middle of the wave envelope.
- In the lateral direction the growth of transverse perturbations affects the highest wave and its first predecessor. Lateral features in the form of standing waves across the (periodic) wavetank appear.

The combination of these two effects gives rise to a fully 3D structure of the wave group. Fig. 4b shows the evolution after just one more wave period, at time $t/T = 91$. The observed wave evolution is clearly a truly directional self-focusing process. Finally, the appearance of curved wave fronts is an important feature of such 3D waves (Fig. 5). These wave groups are characterized by skewed wave patterns that qualitatively agree with Su's experiments.

According to the existing theory, instabilities of type II only affect the steepest waves. The combination of lateral effects with the BF instability, however, has not yet been properly studied in laboratory experiments for the highest waves. Based

(a)

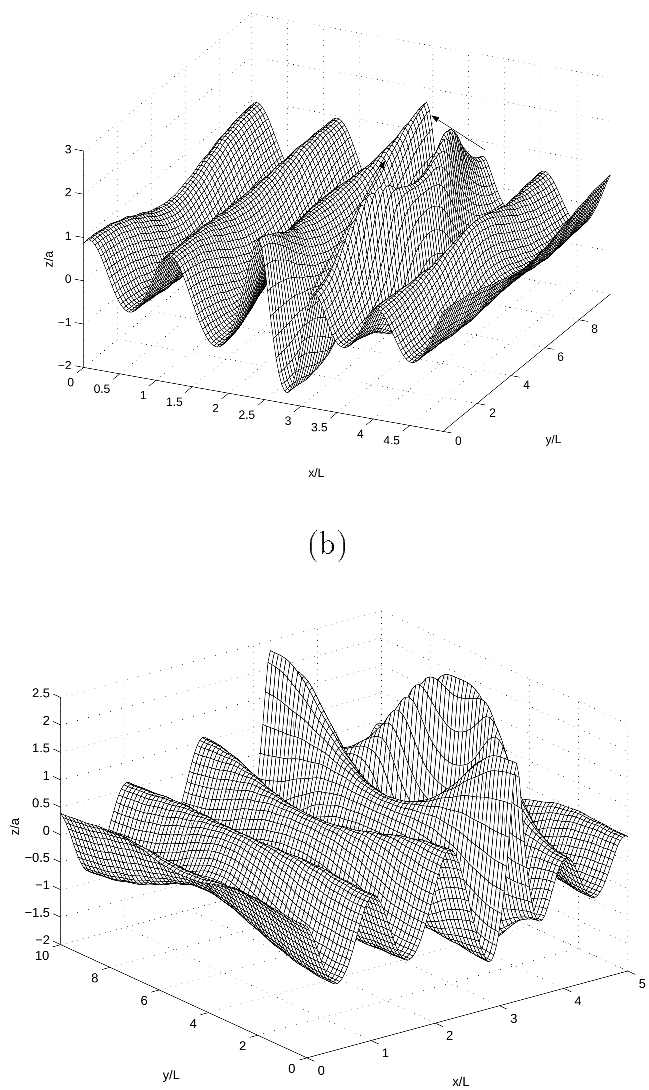


**FIG. 4. Evolution of a doubly modulated wave train after: (a) 90 and (b) 91 wave periods, with $ly = 2lx$**

on our limited computational results, our analysis is that the BF-like mechanism produces a short wave group of increasing height and steepness, and it is within such a group that the lateral instability significantly manifests itself, provided the modulational wavelength in the lateral direction is long enough. In fact, not all longitudinal perturbations produce a BF instability, as well as not all lateral perturbations are able to produce instabilities of type II. For instance, the evolution of a modulated wavetrain having the same initial steepness $ak = 0.14$ and $l_x = 5$, but a shorter $l_y = 4$ (so that the lateral modulational wavelength is 0.8 times the longitudinal one) is shown in Fig. 6 at time $t/T = 90$. In this case,

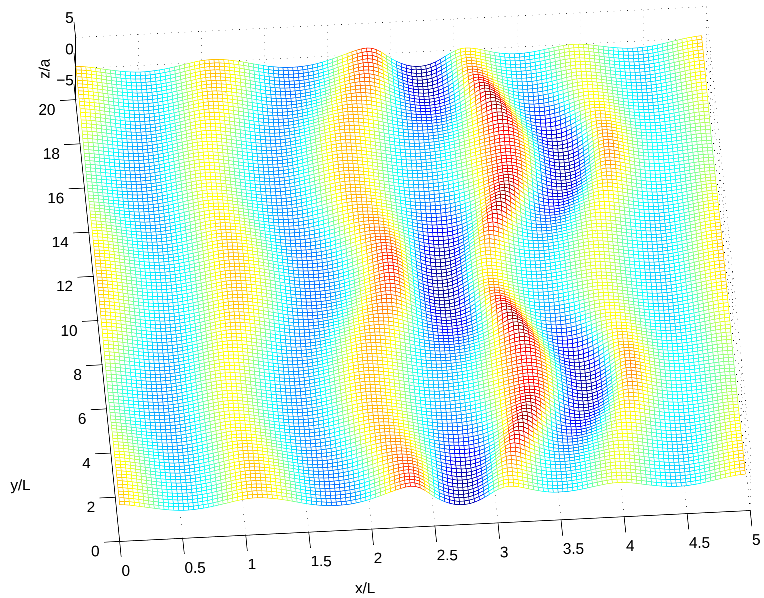


**FIG. 5. Spatial structures of the doubly modulated wave train in Fig. 4**

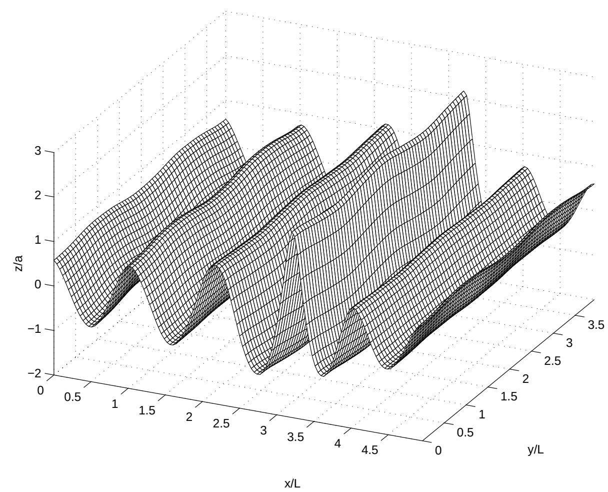


**FIG. 6. Evolution of a doubly modulated wave train after 90 wave periods,** $ly = 0.8lx$

only the longitudinal modulation grows significantly, according to a classical BF modulational mechanism. The modulation growth observed in 3D modulations should be limited by wave breaking, which cannot be described by a single-value free surface representation such as used in the HOS method. Breaking will not happen uniformly along a wave crest, and a 3D self-focused breaking wave is expected to appear at some stage of the modulation.

## CONCLUSIONS

We simulated fully nonlinear 3D focused and self-focused waves, in nonlinear wave models, with the goal of understanding the kinematics and dynamics of 3D large transient waves, as they occur in nature.

In future work, the computationally efficient HOS method could be used to calculate the initial stages of the self-focusing modulation (i.e., the longer duration ones, on the order of 100 wave periods). Then, free surface elevations and potential $[\eta(x,y,t), \phi(x,y,t)]$, found in the HOS solution, could be used to initialize a 3D-NWT having doubly periodic boundary conditions specified on lateral boundaries.

Therefore, 3D self-focusing cases producing extreme, possibly breaking (the worst scenarios for engineering applications), waves could be studied in the NWT. This would be quite difficult to do in a laboratory, due to the long distances of propagation required for the instabilities to grow.

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