Evolution of three-dimensional unsteady wave modulations

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Abstract. A numerical investigation of nonlinear interaction mechanisms producing large wave energy concentrations, which lead to episodic transient waves, is performed using both a Higher Order Spectral (HOS) model and a three-dimensional (3D) fully nonlinear Numerical Wave Tank (NWT). Self-focusing of wave energy is achieved through modulating a periodic wave train along two orthogonal directions. Nonlinear unsteady 3D wave groups are obtained, which show a curved wavefront structure, with focusing of wave energy in both the directional and the frequency domains. Breaking would ultimately occur in such groups. This, however, cannot be described by the HOS model but, based on the HOS solution, both breaking and non-breaking freak waves may be simulated in the NWT, and their shape and kinematics can be studied.

1 Introduction

A number of attempts have been reported in the literature to produce freak waves by nonlinear self-modulation of a two-dimensional slowly modulating wave train. Both solutions based on the (weakly) nonlinear Schrödinger equation (NLS), or its modifications [4], and numerical models solving fully nonlinear free surface flows, have been proposed [5, 17]. Freak waves have been observed to be essentially three-dimensional (3D) phenomena. McLean [8] theoretically predicted a type of wave instability (called type II), which is predominantly 3D, in contrast with the 2D instability (type I; i.e., the side-band instability) identified by Benjamin and Feir (BF) [7], which leads to the formation of wave groups in quasi-2D swells, through a self-focusing mechanism. Su *et al.* [10] experimentally confirmed this prediction by showing how a steep 2D wave train can evolve into 3D spilling breakers. Hence, 3D modulational instabilities cannot be neglected when describing the steepest ocean waves. Two-dimensional nonlinear wave instabilities have been simulated in a few numerical studies, by slow self-modulations of a 2D periodic wave train (Dysthe and Trulsen [4]; Henderson *et al.*, [5]). In such studies, an initially periodic wave train of moderate steepness is perturbed by a small periodic perturbation. After a large time of propagation (typically over 100 wave periods), it is observed that a large steep wave, i.e., a freak wave, may emerge from the initial wave train, and break or recede and periodically reappear. In these studies, 3D effects were not usually addressed because, either it was not possible to generalize the method of solution to 3D, or the computational effort in a 3D model was too high. Nevertheless directional effects are of prime importance. Breaking may occur, when waves reach a sufficient size, at some stage of the modulation. Nepf *et al.* [14], for instance, experimentally showed that curved wave fronts lead to 3D breaking in ocean waves, and that the shape and kinematics of 3D breaking waves greatly differ from those of two-dimensional (2D) breakers (see also She *et al.* [15]; and Johannessen and Swan [19]). The degree of angular spreading is found to have large effects on wave breaking characteristics and kinematics, and non directional wave theories are demonstrated to be insufficient to describe the kinematics of 3D waves.

Since many extreme (freak) waves are expected (and have been observed) to be 3D, modulational instabilities occurring in three dimensions cannot be neglected when describing the steepest waves.

2 Three-dimensional modulations

The computationally efficient Higher Order Spectral (HOS) method [17, 18] is used in the present computations, assuming doubly periodic boundary conditions in the computational domain. Extreme waves are produced through the evolution of 3D wavetrains subjected to both longitudinal and lateral modulations. Modulations of this type are characterized by the initial steepness of the wave train (ak), and by two characteristic wavelengths, for the longitudinal and transverse modulations, respectively.

A transverse modulation is superimposed to the longitudinal one. For the free surface elevation, this leads to expressions of the form,

$$\eta_p = 2\mu\,(ka)\cos\left(2\pi\frac{y'}{\lambda_y}\right)\cos\left(2\pi\frac{x'}{\lambda_x}\right)\cos\left(kx-\omega t\right) \tag{1}$$

where a is the wave amplitude, k the wavenumber, ω the circular frequency of the initial 2D wave train (which, here, for sake of illustration, is simply sinusoidal), and λ_x and λ_y are the longitudinal and transverse wavelengths of the perturbations, respectively (in terms of the longitudinal wavelength $\lambda = 2\pi/k$ of the initial 2D wavetrain; dashes indicate nondimensional variables).

A systematic study of such kinds of modulations, would require, for each wave steepness, the evaluation of the influence of both the longitudinal and the transverse wavelengths, on the evolution of initially slowly modulated wavetrains.

The evolution of a modulated wavetrain having $ak = 0.14$, $\lambda_x = 5$ and $\lambda_y = 10$ (hence with a lateral modulational wavelength that is two times the longitudinal one) is described in Figs. 1-4. The initial regime shown in Fig. 1 is composed of nearly uniform Stokes waves. At these early stages of the evolution, waves are essentially 2D while, at later stages, the growth of transverse

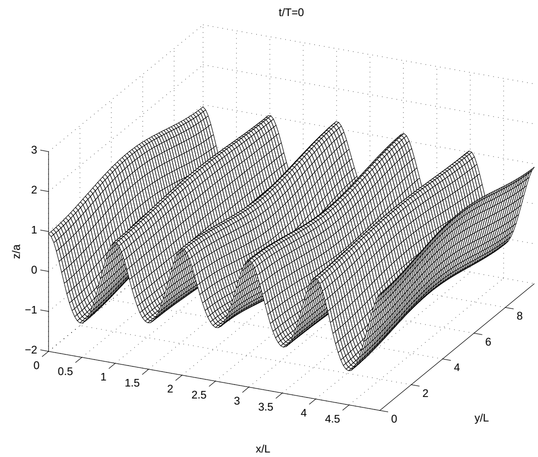

Fig. 1. Initial condition for a 3D wave train modulated in both the longitudinal and the transverse direction

perturbations causes a 3D structure to develop. At final stages of evolution, both a longitudinal and a transverse growth of modulations are observed. Fig. 2, for instance, shows the evolution of the wavetrain at time $t/T = 90$. This evolution results from the combination of two effects :

1. In the longitudinal direction a BF-like mechanism causes the wave group to shorten ahead and to lengthen behind, with a wave energy concentration in the middle of the wave envelope.
2. In the lateral direction the growth of transverse perturbations affects the highest wave and its first predecessor. Lateral features in the form of standing waves across the (periodic) wavetank appear.

The combination of these two effects gives rise to a fully three-dimensional structure of the wave group. Fig. 3 shows the evolution after just one more wave period, hence at time $t/T = 91$; we see that a large crest elevation is produced. This clarifies the evolution as a truly directional self-focusing process. The 3D structure of this doubly modulated wave is more evident in planview (Fig. 4), where an identical wavefield has been placed at one of the lateral sides (this is possible because of the lateral periodicity assumed for the computational domain). The appearance of curved wave fronts is an important feature of such 3D waves. These wave groups, as shown in Figs. 2-4, are also characterized by skewed wave patterns that qualitatively agree with Su's experiments. In particular :

1. The system of oblique wave groups, which is seen to radiate symmetrically from the primary wave direction, seems similar to that observed in the experiments. The angle, locally measured in these oblique wave fronts, approaches the 30^o value which was found experimentally.
2. A shifting of the lateral wave forms between two consecutive rows.

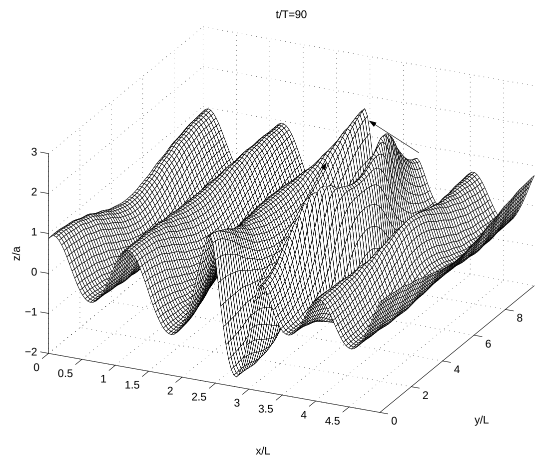

Fig. 2. Evolution of the wave train in Fig. 1 after $t/T = 90$

Our interpretation of these observations is that the BF-like mechanism produces a short wave group of increasing height and steepness, and it is within such a group that the lateral instability manifests itself, if the modulational wavelength in the lateral direction is long enough. For instance, the evolution of a modulated wavetrain having the same initial steepness $ak = 0.14$ and $\lambda_x = 5$ $\lambda_y = 4$ (so that the lateral modulational wavelength is only 0.8 times the longitudinal one) is shown in Fig. 5. In this case, only the longitudinal modulation grows according to a classical BF modulational mechanism. More details can be found in [3].

The growth of perturbations leads, for the steepest initial waves, to a rapid development of high wavenumber instabilities. A few time steps later, the model fails to converge. Using the HOS method, it is not possible to conclude whether this would be a case leading to wave breaking, but the range of wave steepness over which such numerical instabilities occur is consistent with typical values of steepness, relative to the occurrence of spilling breakers observed in laboratory experiments $(ak > 0.25)$.

To be able to follow the evolution of this system further in time, after numerical breaking occurs, an ideal filter, removing all high frequency components and producing a loss of energy, has been applied. In this case, the loss of one or two wave crests may occur after the wavetrain has reached the maximum stage of modulations. This effect is the equivalent of the downshifting observed in physical experiments. A similar tendency to lateral energy transfer is also reported by Trulsen and Dysthe [11], who suggested that the full explanation of this downshift probably involves the combined effects of 3D nonlinear modulations, dissipation, and wave breaking.

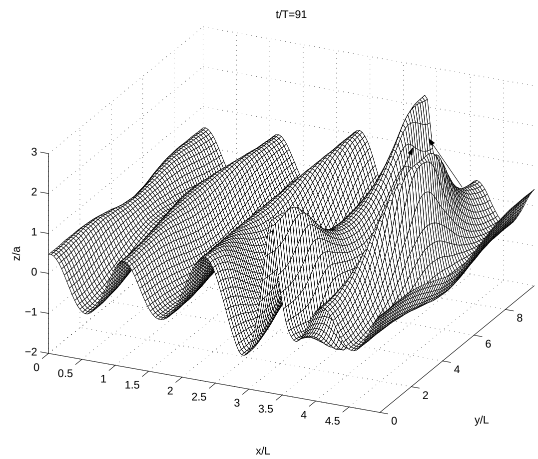

Fig. 3. Evolution of the wave train in Fig. 1 after $t/T = 91$

3 Three-dimensional breaking waves

The modulation growth observed in 3D modulations should be limited by wave breaking, which cannot be modeled using a method describing the free surface as single-valued, such as the HOS method. Breaking will not happen uniformly along a wave crest, and a 3D self-focused breaking wave is expected to appear at some stage of the modulation. A 3D fully nonlinear potential flow model, with an Eulerian-Lagrangian flow representation, recently developed by Grilli et al. [16], has been extended to represent 3D directional and wave focusing, including the additional possibility of frequency focusing such as studied in earlier 2D nonlinear models. To do so a "snake" wavemaker similar to those used in laboratory facilities is modeled at one extremity of a 3D Numerical Wave Tank (NWT), while a snake absorbing piston is modeled at the other extremity of the NWT to minimize the effect of wave reflection. Details can be found in [2]. In directional focusing, waves are focused in front of the wavemaker. For instance Fig. 5 shows an example of directional wave focusing where waves are focused at a distance $x_f = 2\lambda$ in front of the wavemaker. In Fig. 6 a case with more intense directional energy focusing is shown, producing a giant steep wave a short distance away from the wavemaker, whose crest is starting to break by spilling breaking.

Very large, possibly breaking (i.e., overturning), 3D transient waves could be modeled in this 3D-BEM model, by using the HOS method to compute the first stages of growth of wave modulations (the longer ones, on the order of 100 wave periods) as initial condition for the model. In this case the initial wave elevation and velocity potential are specified at time t on the free surface, based on the HOS solution. This will be the object of further studies.

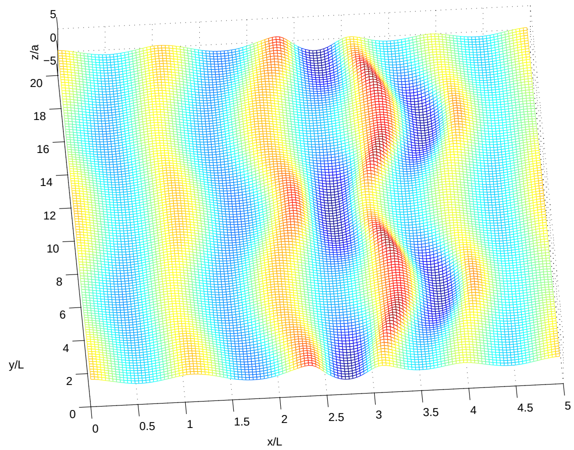

Fig. 4. Planview of the situation depicted in Fig. 3 clearly showing the appearance of curved wave fronts.

4 Conclusions

Three-dimensional self-focusing of wave energy is achieved through modulating a periodic wave train along two orthogonal directions. Both the well known Benjamin-Feir instability (essentially 2D) and 3D instability mechanisms are found to be important for describing the evolution of nonlinear waves. Nonlinear wave interactions produce an instability which transforms an initially two-dimensional wavetrain into a three-dimensional unsteady wave pattern, with short-crestedness in the lateral direction. When the transverse modulation wavelength is sufficiently large, one can observe the growth of the lateral modulation through the absorption of part of the longitudinal wave energy. The model not only predicts the initial stages of instability, but also the evolution of unsteady modulations of 3D finite amplitude waves in a fully nonlinear sense. Three dimensional effects lead to the natural formation of locally curved wave fronts which spread energy from the primary (longitudinal) motion to the secondary (transverse) one. This curved structure of 3D wave groups produces a self-focusing mechanism in both the directional and the frequency domain. Ultimately, this would lead to wave breaking, which cannot be described by the HOS model. However this 3D self-focusing case can be studied in the NWT, which has the capability of modeling both breaking and non-breaking freak waves. Such as study would be very difficult to achieve in a laboratory tank, due to the long distances of propagation required for both the 2D and 3D instabilities to grow.

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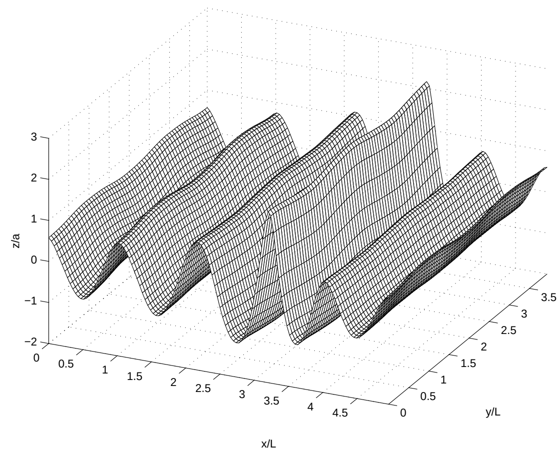

Fig. 5. Evolution of a 3D wave train ($ak = 0.14$, $\lambda_x = 5$, $\lambda_y = 4$) after $t/T = 90$. Note the differences with the corresponding evolution of the wavetrain ($ak = 0.14$, $\lambda_x = 5$, $\lambda_y = 10$ represented in Fig. 2, at the same stages

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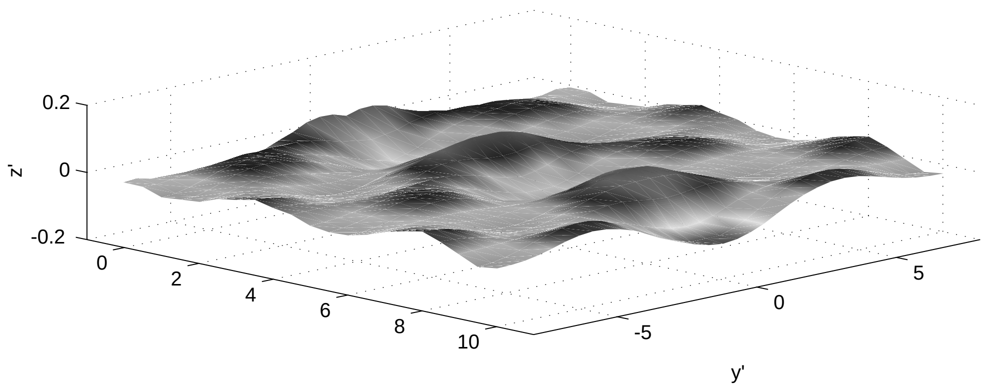

Fig. 6. Directional wave energy using a snake wavemaker in a 3D-NWT. Non breaking case

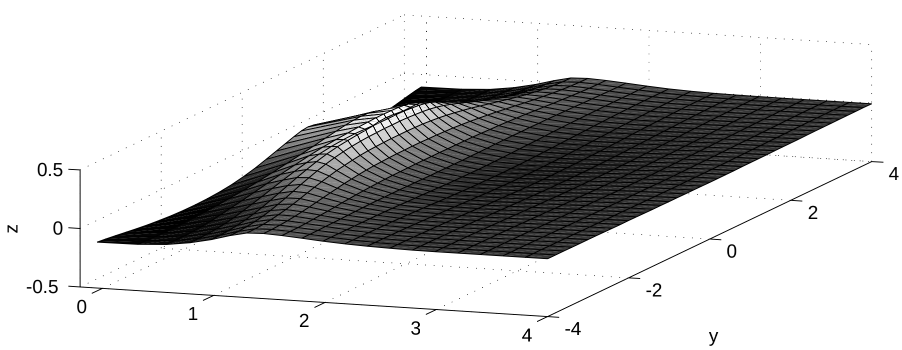

Fig. 7. Directional wave energy using a snake wavemaker in a 3D-NWT. Breaking case

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