PHASE SPACE WARPING BASED MULTI-MODAL DAMAGE IDENTIFICATION: 
AN EXPERIMENTAL STUDY

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ABSTRACT

In this paper, a multi-dimensional damage identification scheme developed in previous work is validated experimentally. The experimental apparatus consists of a driven two-well magneto-elastic oscillator, where the magnetic potential is perturbed by two electromagnets. These electromagnets are activated by a computer controlled power supply and their terminal voltages are considered a two-dimensional damage variable. The effect of total change in the supply voltage of the electromagnets is so small that it causes maximum 2% shift in the experimentally measured natural frequencies of small oscillations in each well of the potential. The experimental run is started in a nominally chaotic regime. The battery voltages are altered on specific trajectories in the damage (voltage) phase portrait. The elastic vibration data is collected using a strain gauge and an accelerometer mounted at the base of a clamped beam and near its free end, respectively. Both data sources are used separately to calculate tracking metrics based on the phase space warping concept. The damage identification is achieved by applying a smooth orthogonal decomposition to the calculated statistics. The reconstructed battery phase space trajectories based on both data sources show affine relationship with the original control trajectories.

INTRODUCTION

In engineering systems, gradual component deterioration (e.g., fatigue damage) leads to imminent failure. The high demand for safety and reliability stimulates critical interest in developing damage diagnosis technology. However, complexity of modern systems makes the diagnosis extremely hard, especially when several damage mechanisms are simultaneously present in a system. There are wide variety of solutions proposed in the existing literature, however most of them focus on diagnosis of some special scalar damage processes. In our previous work, a multi-dimensional damage identification method [1] has been developed based on Phase Space Warping (PSW) [2]. This method has been verified using simulation of numerical models [1]. Here, an experiment is designed to simulate the evolution of two independent damage processes, and our approach is applied to identify the two-dimensional damage.

In the next section, a brief literature survey is provided to sketch the current state-of-the-art in damage identification utilizing systems’ nonlinear properties. A sketch of our damage identification approach is also provided for completeness. Then the experiment system is introduced, and the damage identification method is applied to the collected data. After that we compare the performance of damage identification results from different data sources. Finally, we conclude with discussion of results.

BACKGROUND

There are several good review articles available, which cover the diverse field of damage identification and diagnosis. For example, Sohn et al. [3] and Zou et al. [4] provide extensive discussion of developments in this field. In [1] we have also provided general discussion of available literature. Here, we give a
brief overview of some new developments to provide the general background for presented work.

Systems' nonlinear features for damage detection attract more attention from researchers in recent years. Brandon [5] stated that nonlinear identification can provide valuable information for structural health monitoring. Wang, et al. [6] show that system's chaotic response provides intrinsic information on the underlying dynamical system, and applied several nonlinear time series analysis approaches to indicate the existence of certain damage in a large rotating machinery. Douglas et al. [7] proposed that nonlinear identification can provide valuable information on extracting long-term invariant features from measured time series to detect damage. However, systems with damage accumulation are usually structurally unstable—drifting system parameters cause bifurcations in steady state behavior. Therefore, long-term invariant features are not expected to be a smooth functions of drifting parameters and are unsuitable for continuous damage tracking.

In our previous work [2, 9, 10], a novel damage identification and diagnosis method has been developed in a dynamical system framework [11]. A linear one-to-one relationship between tracking vectors and actual damage states has been demonstrated. This approach has been successfully applied to one-dimensional damage identification problem [2, 10]. By introducing new feature vectors, and Smooth Orthogonal Decomposition (SOD) [10], the method has been extended to solve multi-dimensional damage identification problem.

OVERVIEW OF DAMAGE IDENTIFICATION METHOD

Dynamical system approach views damage as a slow-time state variable evolving in a hierarchical dynamical system, where a fast-time system dynamics is coupled with a slow-time damage evolution. Based on this approach, PSW-based metrics are used to provide functional connection from measured fast-time dynamics to hidden slow damage accumulation. For identifying active damage states from the calculated PSW-based metrics we use smooth orthogonal decomposition (SOD). Here a brief overview of main concepts and procedures are given.

Damage Evolution in a Hierarchical Dynamical System

We focus on identifying damage in a class of dynamical systems described by following equations of motion [2, 12]:

\[ \dot{x} = f(x, \mu(\phi), t), \quad (1a) \]
\[ \dot{\phi} = \varepsilon \cdot g(x, \mu), \quad (1b) \]

where \( x \in \mathbb{R}^n \) is a directly observable, fast-time variable; \( \phi \in \mathbb{R}^m \) is a hidden slow-time damage variable; \( \mu \in \mathbb{R}^r \) describes material parameters of Eq. (1a) and is a function of \( \phi \); \( 0 < \varepsilon \ll 1 \) is a rate constant that defines the time scale separation between fast-time and slow-time subsystems; overdots denote differentiation with respect to time, \( t \). Because \( \varepsilon \ll 1 \), the drift caused by damage evolution is negligible over time scales of \( \mathcal{O}(\varepsilon) \), and the fast-time subsystem can be considered to be quasi-stationary for these time scales.

Phase Space Warping

In order to identify and track evolution of multi-dimensional damage, it is necessary to establish connection between measured time series and hidden damage states. In experimental context, the analytical form of Eq. (1) is generally unknown. However, some measurements from Eq. (1a) are available. These, measurements are usually sampled at uniform time intervals, \( t_s \), and here we assume that the available measurements are a smooth scalar function of fast-time variable that effectively couples all active degrees-of-freedom of fast-time dynamics. Therefore, topologically equivalent structure of the extended phase space trajectory of Eq. (1a) can be reconstructed using delay coordinate embedding [13, 14]. The measured scalar time series \( \{x(r)\}_{r=1}^M \) is converted into a series of vectors \( \{y^f(r) = [x(r), x(r + \tau), \ldots, x(r + (d-1)\tau)] \in \mathbb{R}^{d\tau} \}_{r=1}^M \}, \) where \( \tau \) is a suitable time delay, and \( d \) is the appropriate embedding dimension. Both of these parameters can be determined using standard techniques [15, 16].

The reconstructed state vectors are governed by an as yet unknown map of the form

\[ y(r + k) = P_k(y(r); \phi), \quad (2) \]

where \( P_k : \mathbb{R}^d \rightarrow \mathbb{R}^d \) is generally nonlinear. The drift in the damage variable \( \phi \) will cause distortions in the phase space, altering the evolution of trajectories. The phase space warping (PSW) refers to these changes in the vector field. In previous work [12, 17], the following PSW tracking function

\[ E_k(y; \phi) \equiv P_k(y; \phi) - P_k(y; \phi_0) \quad (3) \]

was proposed for damage identification. In Eq. (3) \( \phi_0 \) is the reference or healthy state of the damage variable.

In [12, 17], a suitably averaged \( E_k(\phi; y(r)) \) was successfully used to track a scalar battery voltage variable. It was demonstrated that the relationship between the tracking function and the damage variable can be well approximated by an affine map, under certain linear observability conditions.

To actually calculate the tracking function \( E_k(\phi; y(r)) \) for any given initial condition \( y(r) \), we need to know how the fast
subsystem evolves over the time interval \(kt_i\), for the current value of \(\phi\), as well as how this subsystem would have evolved for the reference value of \(\phi_0\). Since the system’s fast-time behavior for the current value of \(\phi\) is directly measurable (i.e., we can reconstruct the fast-time trajectory using a sensor measurement from the fast subsystem), the strategy is to compare it to the predictions of a reference model describing the fast system’s behavior for \(\phi = \phi_0 + \mathcal{O}(\varepsilon \theta_t)\). Here, local linear models are used as the simplest form of a globally nonlinear reference model

\[
y(r + k) = A_k y(r) + a_k, \tag{4}
\]

where \(A_k\) is a \(d \times d\) matrix and \(a_k\) is a \(d \times 1\) vector. Eq. (4) approximates Eq. (2) for a particular point \(y(r)\) in the reference system’s reconstructed phase space. Note that, in practical applications, other modeling solutions such as neural nets (e.g., [18]) or auto regressive moving averages (e.g., [19]) may be more appropriate. The parameters of local linear models are determined by calculating the best linear fit between \(N\) nearest neighbors of \(y(r)\) and their future states. Then the tracking function (Eq. 3) for the initial point \(y(r)\) can be written as

\[
e_k(\phi; y(r)) = y(r + k) - A_k y(r) - a_k + E_k^M(y(r)) = E_k^M(y(r)) + E_k^N(y(r)), \tag{5}
\]

where \(E_k^M(y(r))\) represents the local linear model error, and

\[
E_k(\phi; y(r)) = y(r + k) - A_k y(r) - a_k \tag{6}
\]

is the estimated tracking function that can be determined experimentally. The use of \(E_k^M(\phi; y(r))\) in place of \(E_k(\phi; y(r))\) is justified, if \(E_k^M\) is small compared to \(E_k^N\).

**Damage Tracking Feature Vector**

In previous work [1, 12, 17], a suitable norm of the averaged value of the estimated tracking function over one data record \(\|E_k\|\) provided a good tracking metric for scalar damage variables. However, as discussed in detail in [17], there generally will be superfluous fluctuations in the tracking results caused by:

(a) the change in the population of points from data record to data record; and (b) the changes in the map of Eq. (2) and model error \(E_k^M\) from point to point.

The population changes are caused by the evolution of \(\phi\) that acts as a bifurcation parameter for Eq. (1a). As parameters of Eq. (1a) traverse their range due to the accumulating damage, the system will undergo changes in its steady state behavior. These changes will be abrupt and can not be used to track \(\phi\), which evolves smoothly. This will also cause change in population of points from record to record. The old tracking metric could only deal with minor changes in population of points that extended the original coverage of the reference population.

Two main sources of superfluous fluctuations, which are not related to changes in \(\phi\) are:

(i) changes in the model fit error \(E_k^M\) from point to point, caused by a nonuniform coverage of reference phase space by the points on a reconstructed trajectory; and

(ii) changes in the actual nonlinear mapping of Eq. (2), also from point to point. In the old metric the fluctuations due to the variability of modeling error were compensated for by a suitable weighted average of \(\|E_k\|\) over one data record, while the fluctuations caused by the change in the mapping of Eq. (2) were not alleviated.

Here, we compensate for all sources of Eq. (2) fluctuation by using a new tracking feature vector. This new approach evaluates the average value of the estimated tracking function in \(N_s\) disjoint regions \(\mathcal{S}_k (i = 1, \ldots, N_s)\) of the reconstructed phase space

\[
e_k^i = \|\mathcal{S}_k\|^{-1} \sum_{y \in \mathcal{S}_k} E_k(\phi; y), \tag{7}
\]

and combines them in one damage tracking feature vector:

\[
S_k = [e_1^k; e_2^k; \ldots; e_{N_s}^k], \tag{8}
\]

where \(S_k\) is a \(dN_s \times 1\) vector calculated for each data record.

The choice of parameter \(k\) depends on the data sampling rate and effect of noise on the sensitivity of the tracking function. The parameter \(k\) also affects the sectioning of the phase space into the disjoint regions, which should be done in such a way as to alleviate the sources of unrelated fluctuations with regards to the previous method. In particular, the estimated tracking function Eq. (6) for the points within the same region \(\mathcal{S}_k\) should have approximately the same sensitivity with respect to damage variable. This would reduce fluctuations due to the change in the mapping Eq. (2), as well as the variation in the linear fit error \(E_k^M\) within one region. With the sectioning of phase space we are also more likely to have several regions that have approximately constant population of points from record to record. Thus, the effect of changes in steady state behavior on the feature vector statistics is also reduced.

It is conjectured based on previous work [1, 12, 17] that there is an affine projection \(\mathcal{H}_k : \mathbb{R}^{dN_s} \rightarrow \mathbb{R}^m\) that maps the proposed feature vector \(S_k\) onto the damage state \(\phi \in \mathbb{R}^m\):

\[
\phi = HS_k + h, \tag{9}
\]

where \(\phi\) is an estimate of actual damage state, \(H\) is an \(m \times dN_s\) matrix, and \(h\) is an \(m \times 1\) vector.
Identification: Smooth Orthogonal Decomposition

In many practical situations we do not have the direct measurement of damage state or means to estimate the affine projection parameters of Eq. (9). Therefore, the damage tracking feature vectors \( S_k \) have to be used directly to determine the observable facts about the hidden damage state. The SOD is a generalization of the optimal tracking method [20] that relies on the existence of underlying deterministic behavior of the damage accumulation process but does not require its model. This method is based on maximizing smoothness and overall variation in the feature vector, found by solving an eigenvalue problem. For completeness we give a brief description of the procedure in the following paragraphs.

The feature vector \( S_k \), Eq. (8), is estimated for \( N_r \) data records. These vectors are arranged in a \( N_r \times N_s \) damage matrix, called \( Y \). Each column of this matrix is normalized by subtracting its mean and scaling it to unit norm. Next, we identify the SOD-based tracking metric by a linear projection of the matrix \( Y \)

\[
\varphi_s := Yc,
\]

that varies smoothly. This can be accomplished by defining \((N_r - 1) \times N_r\) derivative matrix

\[
D := \begin{bmatrix}
1 & -1 & 0 & \ldots & 0 \\
0 & 1 & -1 & \ldots & 0 \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
0 & \ldots & 0 & 1 & -1
\end{bmatrix}
\]

and minimizing the following ratio with respect to \( c \)

\[
q(\varphi_{at}) := \frac{\|D\varphi_s\|^2}{\|\varphi_s\|^2}.
\]

In minimizing \( q \) we maximize the smoothness and overall variation of \( \varphi_s \). The minimum of \( q \) is the smallest generalized eigenvalue \( \lambda \) of the following eigenvalue problem

\[
[(DY)^T DY]c = \lambda [Y^T Y]c.
\]

The eigenvectors corresponding to the smallest eigenvalues give the optimal \( c \) and, hence, \( \varphi_s \). In the experiment two main hypotheses will be tested:

**Hypothesis # 1:** the \( m \) smallest eigenvalues of Eq.(13) will be several magnitudes of order smaller than the rest in the presence of \( m \)-dimensional damage evolution process

**Hypothesis # 2:** the corresponded eigenvectors will be within a linear transformation of the actual damage states.

**DESCRIPTION OF EXPERIMENT**

To test the main hypothesis advanced in previous section and experimental systems was constructed to simulate a two-dimensional damage accumulation process. The experimental apparatus is based on a modified version of the well-known two-well magneto-elastic oscillator described in [2]. The oscillator (see Fig. 1) consists of cantilever beam, which is constrained to one degree-of-freedom motion using stiffeners and is mounted on an electro-magnetic shaker. The nonlinear potential at the beam tip is realized by two permanent/electromagnet stacks, which are powered by a two-channel programable power supply. The supply voltages to the electromagnets are altered using a computer control of the powersupply. A strain gauge and an accelerometer are attached on the hinge and the tip of the beam, respectively, to record the position and acceleration of the beam. The shaker provides harmonic excitation to the mounting frame. The frequency and and amplitude of the excitation are also computer controlled.

Data acquisition and control was conducted using PC workstation equipped with a National Instruments PCI-6052E multi-
channel data acquisition card and LabView programming environment. This PC-based system was used to digitize and record signals from strain gauge, accelerometer, and power supply’s terminal voltages. It was also used to provide a harmonic driving signal to the electromagnetic shaker. The programmable power supply (Agilent E3647A) was controlled through RS-232 and LabView environment.

For the experiment the frame is forced with a 10 Hz harmonic signal. Amplitude of the driving signal is set to obtain nominally chaotic response for a fully loaded batteries. The voltage supplied to the electromagnets is altered harmonically: \( v_1(t) = 9 \cos(2.65 \times 10^{-6}t) + 10 \), and \( v_2(t) = 9 \cos(5.3 \times 10^{-6}t - \pi/2) + 10 \). The voltage updates are done at 1 KHz, for every 1.14 mV change, refer to Fig. 2. The frequency response functions (FRFs) of small oscillations in each energy well were estimated to quantify the overall influence of the change in the supply voltage on the parameters of fast-time system. Fig. 3 and Fig. 4 show the FRFs calculated for the minimum and maximum expected voltage supply for the electromagnets in the front and back energy wells, respectively. From the plots it is apparent that the maximum change in the natural frequency is limited to 2% that corresponds to the maximum 4% shift in the stiffness of the beam in each energy well.

The data from the strain gauge, accelerometer, and power supply terminals are lowpass filtered with 50 Hz cutoff frequency, digitized with 160 Hz sampling frequency and stored on a hard drive. The whole experiment lasts a little over 4.5 hours, and total 2.6 million data points are recorded. The data from strain gauge is plotted in Fig. 5. System exhibits substantial structural instability, which manifests itself from numerous transitions between the variety of observed apparently periodic and chaotic motions. One of these large periodic windows in the data is apparent in Fig. 5 between 3.5 and 4 hours.

**DAMAGE IDENTIFICATION**

The first 215 points of the collected data sets are used for constructing a local linear reference models. The delay time is estimated to be 6\( t_e \) and embedding dimension of 5 is found to be sufficient to ensure unique embedding of the trajectories based on measured scalar time series of both strain and acceleration. The prediction time is set to \( k = 1 \) and the parameters of each linear model are estimated based on 32 nearest neighbors of point of
The strain and accelerometer time series are divided into disjoint data records of $M = 2^{14}$ points in each. Therefore, each time series yields total of 159 data records of size $M$. For estimating the feature vectors $S_1$ for each data record, the phase space was sectioned into $N_r = 16$ disjoint uniform slabs along the third coordinate of the embedded vectors. Time series corresponding to each coordinate of $S_1$ are normalized and assembled into a matrix $Y$ as described in method section.

SOD-based damage identification is applied to the matrix $Y$. The results of SOD applied to matrix $Y$ are shown in Figs. 6–7, which show the calculated generalized eigenvalues of Eq. (13) for strain and accelerometer time series, respectively. By looking at this figures, we cannot make as clear a judgment as with the numerical experiment [1] about the existence of two dominant smooth orthogonal modes. However, there is still some indication that this is the case, especially for the data for strain time series. This can be explained by the limited amount of data that is used to estimate statistics. It is usually customary to use at least 10,000 points per dimension to accurately estimate statistics from a chaotic data. In our case, this requirement would be 50,000 points. However, we have only used 1,024 point for estimating each $e_1$. This is at least one order of magnitude deficiency in the available amount of data. However, even with this limited amount of data we still have indication that there are two main independent factors contribution to the smooth variations on the tracking matrix $Y$. Therefore, first hypothesis about the separation of eigenvalues is validated experimentally.

In Figs. 8–9 generalized eigenvectors or smooth orthogonal modes (SOMs) that correspond to the two smallest generalized eigenvalues from Figs. 6–7, respectively, are shown. From these figures the similarity of identified modes is apparent. However, the modes identified from the strain time series have consider-

ably smaller local fluctuations when compared to the modes for accelerometer time series. This can be explained by the higher level of noise in the accelerometer time series. Since, both time series yield qualitatively similar trends, for further analysis we will use only the SOMs obtained from the stain time series.

The first indication that our second hypothesis is valid is depicted in Fig. 10. Here the first SOM is compared to a linear combination of actual damage states (power supply terminal voltages). In particular, we have plotted $v_1 - v_2$ on top of scaled SOM in Fig. 10. Clear similarity between these two trends is observed. To further investigate the validity of hypothesis #2 the original phase space trajectory (shown in Fig. 11) is compared with the identified phase space trajectory (shown in Fig. 12). It is clear that these trajectories share qualitatively similar trends.
To actually verify the existence of one-to-one affine map between the identified phase space trajectory and the actual trajectory we estimate this map in the list-squares sense and show the results in Fig. 13.

CONCLUSION

In this paper, we have presented an experimental confirmation of a multi-dimensional damage identification procedure that was reported in [1]. The experiment was designed to introduce a two-dimensional slow-time drift process into a fast-time dynamics of a one degree-of-freedom driven oscillator. The system was a modified version of the two-well magneto-elastic oscillator as studied by [17]. Slow-time damage accumulation was introduced by powered electromagnets that perturbed the potential field of the fast-time system. The effect of this perturbation was a maximum of 4% change in the stiffness of the beam in each potential well.

The vibration of the driven cantilever beam was recorded using a strain gauge and an accelerometer. The SOD-based damage identification scheme was able to identify a two-dimensional damage process. The corresponding SOMs were shown to be in approximately linear one-to-one correspondence with the actual
damage state that were measured independently. Namely, it was demonstrated that the identified battery phase space trajectories show affine relationship with the original control trajectories.

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